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RELIABILITY VARIATION ANALYSIS FOR SPACE SYSTEM DEVELOPMENT

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PART I

GENERAL CONSIDERATIONS

NEW RELIABILITY CONTROL TECHNIQUES

The reliability analysis techniques presented in this report were designed to strengthen management control over reliability aspects of NASA research and development programs. They represent a fresh approach to the problems involved in planning and administering reliability programs for complex hardware developments--an approach with a high potential for reducing reliability costs and increasing assurance of end-product reliability.

Since this new approach hinges upon consideration of failure-rate variability, the techniques derived from it will be referred to, collectively, as "Reliability Variation Analysis" (RVA) techniques.

RVA techniques furnish a practical method for determining, in quantitative terms, the reliability risks associated with a hardware development program. These reliability risks can be used in program planning in much the same way that time and cost data are used in current methods of management planning, particularly the PERT type of program planning; that is, they can be used to establish the critical reliability path and to determine the trade-offs between reliability, time, and cost.

The new analytical techniques provide a method for constructing test programs to derive maximum utility from the available test money, time, and facilities. The methodology for test-program construction can also serve as a criterion for comparative evaluation of proposed testing programs. Moreover, the RVA techniques make possible the integration of test results into the risk-determination process, thus improving the precision of the risk determination and identifying potential reliability problems. Identification of problem areas permits redirection of program activity as required to ensure attainment of the reliability goal for the completed system.

Created in specific response to a serious need for improved planning and control of NASA spacecraft development programs, the RVA techniques are capable of extension to other types of development programs within NASA and in industry. In time, they could become part of the contractual requirements on new research and development programs. At present, most DoD and NASA development programs require the application of established reliability analysis techniques by the contractor. The RVA techniques are not intended to replace these conventional techniques, but to complement them.

RELIABILITY VARIATION ANALYSIS

The two basic techniques which are the foundation of reliability variation analysis are:

1) Risk Determination

2) Resources Allocation for Testing.

The technique of risk determination is used to establish the reliability status of the hardware at each step in the development program. It provides a continuous measure of the chance of system failure, identifies the sources of unreliability, and pinpoints high-risk items.

The technique of resources allocation for testing is used for the construction, evaluation, and revision of the test program in successive stages of system development. It permits selective use of the time, money, and facilities available for testing.

Effective use of these techniques as practical aids in the planning and control of hardware development and production programs requires engineering judgment and a thorough understanding of the underlying concepts.

RISK DETERMINATION

The reliability goal for a new system is established on the basis of the intended use of the system and the current state of the art. However, until a hardware system is tested in operation, there can be no certainty that it will meet its reliability goal. The degree of uncertainty--i. e., the chance that the completed system will fail to meet its assigned goal--is the reliability risk. This risk should be known at each step in the development program in order to redirect program activity as necessary to lessen the chance of system failure.

The reliability risk is composed of two constituent types of risks:

1) Risks assumed in using component parts of known, marginal reliability.

2) Risks incurred in using new parts whose reliability has not yet been established.

In conventional reliability analyses, only risks of the first type are considered. No formal distinction is made between parts of known reliability and parts that are relatively untried; consequently, the two types of parts are given equal weight in arriving at an estimate of system reliability. RVA techniques, on the other hand, provide for recognition of the additional element of risk attributable to lack of information on the real failure rates of parts and larger assemblies of the system--particularly those parts and assemblies which have not been fully tried by practice.

There are many reasons why the failure rates of the constituent parts of a system cannot be known exactly. For example, the failure rates of parts of the same generic type can vary from vendor to vendor, or even from lot to lot obtained from a single vendor. Similar parts used in different applications in the system can have different failure rates due to differences in operating environments. Of two identical units, one may be damaged in the assembly process, etc.

In conventional analyses, failure rates are treated as known, fixed values. In variation analyses, failure rates are described by statistical distributions. It should not be inferred that failure rates are assumed to vary within the system; this technique simply recognizes that the failure rates cannot be known exactly and must therefore be described by probability distributions. The characteristics of the distributions reflect the information available on the failure rates of the parts, units, assemblies, etc., and finally on the failure rates of the system. When the failure rate of the system is described by a statistical distribution, it is possible to compute the reliability risk--i.e., the probability that the system failure rate will exceed the rate corresponding to the reliability goal.

In using the RVA method, the reliability analyst is required to examine each of the sources of failure-rate variation and, on the basis of his factual knowledge and engineering judgment, to establish the magnitude of the variation to be expected from each source. As a guide in this examination, he uses a checklist designed to cover four areas that contribute to failure-rate variation:

1) Part-procurement practices, including incoming inspection controls.

2) System functional design.

3) System packaging design.

4) Assembly procedures, including consideration of workmanship skill.

The reliability analyst assigns a numerical weight to each of these four factors. The weight assigned to a factor reflects his judgment concerning the contribution that the factor can be expected to make to failure-rate variability in the system of interest. That is, the weight indicates whether the contribution of the factor can be expected to be normal, greater than normal, or less than normal. By a simple computation procedure, these weights are used in estimating the average failure-rate value for each unit of the system and in determining the variability possible in the estimate.

As the development program progresses, the reliability risk of the system will change. Computing the new reliability risk is a relatively simple matter. If the system is altered by design changes, the modified portions of the system are re-evaluated and the original estimates are replaced by the results of the new analysis. If the system or portions of the system undergo testing, a simple calculation technique effects the appropriate modifications in the estimates of failure rate and failure-rate variability. By these means, the reliability risk can be updated and made to reflect the current state of knowledge about the system at each successive step in the development program.

If the reliability risk becomes excessive, management must decide on corrective action to improve the chance of system success. A number of alternative courses of action must be considered: redesign of critical portions of the system, use of high-reliability parts, better control of assembly techniques, etc. An alternative that is particularly appropriate when the risk is largely due to lack of failure-rate information is increased testing of the system or portion thereof. The RVA technique of risk determination can be used to pretest the effects of each of the alternative courses of corrective action by estimating the decrease in risk to be expected as a result of their adoption.

RESOURCES ALLOCATION FOR TESTING

The resources allocation technique developed in this report is optimum, in that it furnishes the maximum information obtainable about the reliability of the system for a given expenditure of money, time, and test facilities. This method of allocating testing resources will be most important in the initial planning stages of the program when the test program is constructed, but can also be useful at any step in the program when the reliability risk determination indicates that either more or less testing is needed.

A requisite for optimum allocation of test resources is a method of measuring the amount of reliability information available concerning each portion of the system and of estimating the amount of information that can be gained by testing. In developing this technique, the coefficient of variation of the failure-rate distribution was selected as a figure of merit for measuring failure-rate information.

The coefficient of variation is derived from the mean and variance of the distribution. The mean represents the best estimate of the failure rate of the item, and thus indicates how critical the item is to the system; the variance indicates the degree of accuracy of the estimate. The coefficient of variation combines the two types of information and thus can serve as a numerical index to the total information available on the reliability of the item. The lower the coefficient of variation, the greater the amount of information available.

With this method of measuring reliability information, resources allocation for testing is a linear programming problem. The constraints on time, money, and numbers and types of items available for test are considered against the estimated information returns from various combinations of unit, assembly, subsystem, and system tests. The mathematical formulation is such as to give greater weight to the testing of newly developed portions of the system than to proven, off-the-shelf items.

POTENTIALITIES OF RVA TECHNIQUES

The RVA techniques of risk determination and resources allocation for testing can be applied repeatedly at each major step in the development program, from initial planning to final system testing. Together, they furnish both status information and planning information, and are thus a basis for program modification to achieve optimum system development.

RVA provides a method of determining and reporting the reliability risks associated with the system under development, both in the initial planning stage and at each major step in the development program. Each source of unreliability is identified, and each protion of the hardware system is evaluated in terms of the reliability risks inherent in its development. High risk items are readily pinpointed by the analysis, and potential problems are identified in time to permit corrective action.

A distinctive feature of RVA is a methodology for test-program construction and modification that permits selective use of the available test resources to increase confidence that the completed system will meet its reliability requirement. The selection method permits assignment and reassignment of test resources as required to obtain optimum scheduling and maximum reliability information.

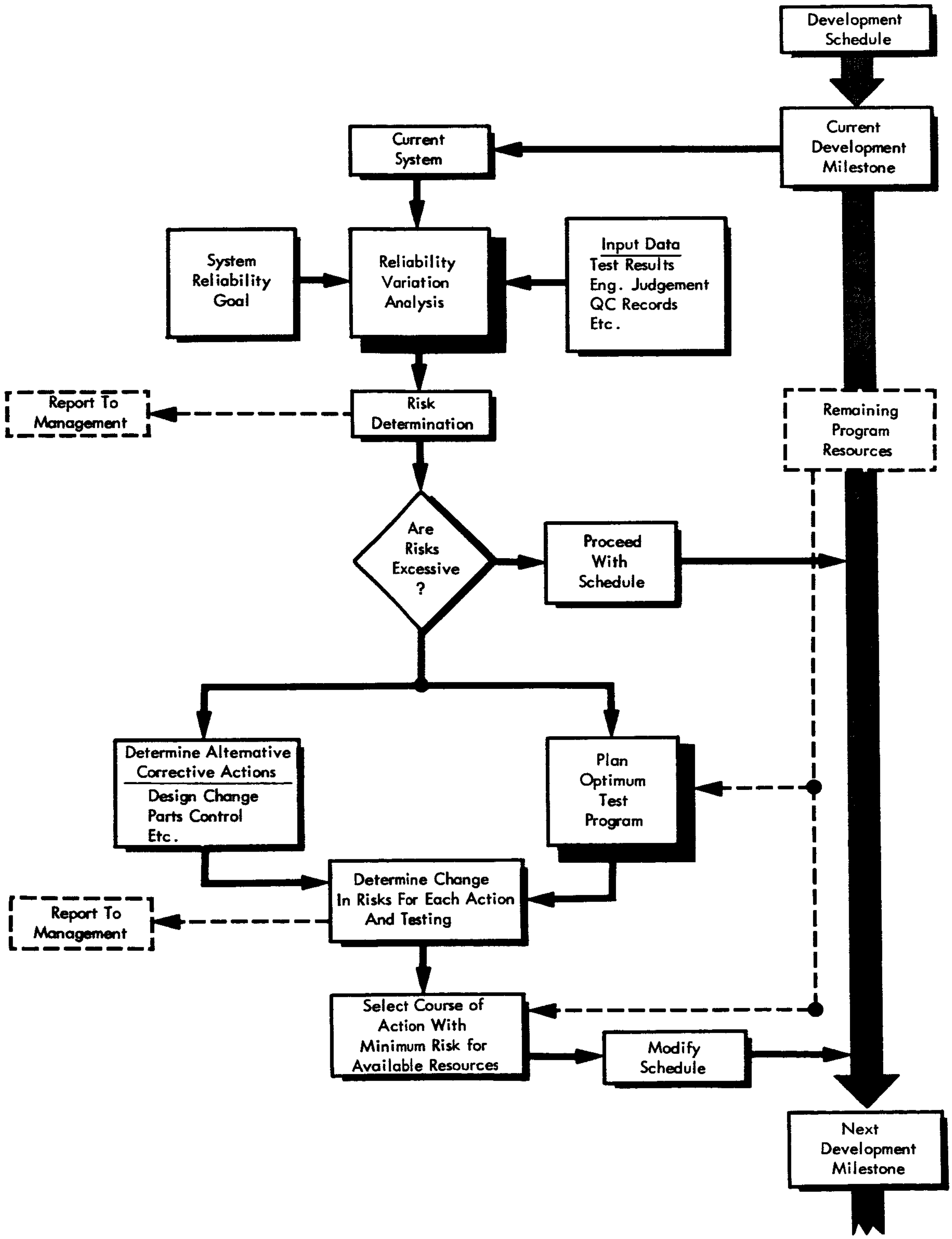
RVA can be applied to most development and production programs, although its values are most apparent in limited-quantity development programs. It is most effective on new, complex hardware developments involving many reliability risks, some of an unknown nature, and on developments which must meet high reliability goals within severe limitations on time and money.

INTEGRATION INTO PROGRAM PLANNING AND CONTROL

Figure 1 is a flow diagram showing how RVA techniques would be integrated into a development program schedule as a control loop between two major milestones of the program. At the completion of a major program milestone, the reliability analyst would determine the reliability risk of the system at that time, using as inputs all available information on system design, test results, quality control data, and engineering judgment. The risk analysis would be furnished to management.

If the risks are not considered excessive, the program would proceed as previously scheduled. If the risks are excessive, an optimum test program would be planned to derive maximum benefit from the remaining program resources: manpower, facilities, quantity of test items, time to scheduled completion, and money.

FIGURE 1
Integration of RVA Control Loop into Program Planning and Control



At the same time, alternative corrective actions, such as design changes, part improvement, etc., would be considered. The reduction in reliability risk expected for each alternative corrective action and for the optimum test program would be computed and submitted to management. Management would then decide which action to take by weighing the cost of each action against the expected lessening of reliability risk. At the next development milestone, the process would be repeated.

UTILIZATION IN DEVELOPMENT PROGRAM CONTRACTING

The methodology for test-program construction and evaluation provided by RVA techniques could also be utilized to the mutual advantage of NASA and potential contractors in the pre-award stages of many future spacecraft programs.

One of the bidder's most difficult problems in preparing a proposal for hardware development is that of planning and pricing a comprehensive test program. The problem is particularly acute for the bidder who proposes to develop and deliver a complex system comprising many components and subsystems supplied by subcontractors whose test activities must be integrated into the overall test plan. Requirements for numerous sequential reliability demonstration tests, even when truncated plans are selected, contribute to the uncertainty in predicting test costs.

Consequently, NASA personnel responsible for evaluating the proposals are faced with a wide variety of test programs and correspondingly wide variations in estimated costs. Frequently, bidders responding to a Request for Proposal conceive testing approaches so individual that they defy direct comparison by the evaluator. If all potential contractors were required by the RFP to utilize the technique for allocation of reliability testing effort described in this report, NASA's task of rating each bidder on a technically sound basis would be greatly alleviated.

The RVA technique for test-effort allocation could be employed in the pre-award phase of a program in the following manner:

> The concepts and details of the technique (perhaps with an example of its application, such as is given in Appendix B) would be included with the RFP. The bidder would be required to make a preliminary assessment of system reliability on the basis of his proposed design concepts and accepted parts failure rates. He would then allocate the test effort in accordance with the RVA technique. Test effort would be directed toward demonstration of achievement of the stated reliability goals, in compliance with the specified level of reliability risk. Individual

tests comprising the plan would be detailed and priced separately. Contingency costs would be identified and allotted to individual tests as deemed necessary by the bidder. Thus, upon completion of the overall test plan, all effort proposed to demonstrate compliance of the delivered hardware to the RFP requirements would be presented in detail. The consistent methodology and detailed presentation would facilitate evaluation of each bidder's approach and comparison of the various proposed test plans.

It is not suggested that the RVA technique for test-program construction be incorporated in the next round of RFP's issued by NASA. As stated elsewhere in this report, the technique is not fully developed and proven. Obviously, if this method of planning and pricing a test program is to be specified in the RFP, it must be followed through the duration of the program. Only by testing the technique on data from completed programs and applying it, simultaneously with conventional methods, on portions of current programs can the practical details of its use be developed. Potentially, however, the technique is a promising tool to assist in evaluating the proposed test plans of bidders on hardware phases of future spacecraft programs.

LIMITATIONS AND CAUTIONS

Reliability variation analysis is not yet a perfected approach to planning and control of the reliability effort, and should not be so represented. It still requires development as a tool and verification in practice. Moreover, it has certain limitations that must be recognized. Among these are its mathematical basis, the quality of the input information, and the difficulties that may be encountered in its application to complex problems. Each of these limitations should be examined in terms of the implications involved.

1. Mathematics

Some mathematicians may be critical of the simplified reliability model used in reliability variation analysis and of the assumptions made concerning the distributional form of reliability variation.

In developing RVA, an effort was made to avoid mathematical complexity. Experience has shown that highly sophisticated techniques frequently fail in practice because of the problem of training people to use them properly. RVA is basically a practical tool, usable largely because it is less complex than a perfect mathematical model.

At present, the distributional form of reliability variation used in RVA can be defended only on the grounds that it appears reasonable. In the future, the distributions should be tested against empirical evidence and sensitivity analyses should be performed to determine the criticality of the distributional form to final results.

2. Input Data

Like other reliability analysis techniques, RVA can be no better than the input information used. The inputs for RVA include the data required to estimate failure rates and failure-rate variability. However, complete certainty in these estimates is not a requirement for satisfactory application because RVA was developed on the premise that exact failure-rate values cannot be known.

Other required inputs are estimates of test costs, test time, and test quantities. At this time, it is not known how precise these estimates must be to yield valid results.

3. Complexity in Application

The implementation of any planning and control technique can be expected to reflect the complexity of the program to which it is applied. With very complex programs, the manipulations of RVA will become complex, leading to complicated reports and difficulties in determining the trade-off of resources. These problems stem, however, from the complexity of the program itself; they are not inherent in the technique employed.

Care should be exercised in initiating the use of reliability variation analysis in development programs. As a new method, it will require more attention and study than well-known and tried techniques. RVA should be applied selectively. If it is used without discretion, the results may be marginal and could undermine confidence in the approach.

RVA should be integrated with existing methods of reliability planning and control. In its development, care was taken to make it compatible with current management control techniques, particularly the PERT type of program planning. It must be emphasized that RVA is not an automatic technique. It is simply a device for integrating factual information and human judgment in a logical and consistent manner. It is no substitute for management decision, but does provide management with information useful in arriving at decisions.

The yield from reliability variation analysis can be significant. As experience is gained, valuable extensions of this approach should be expected.

PART II

THE TECHNIQUES

NATURE OF THE PROBLEM

This part of the report is concerned with perfecting certain existing reliability techniques by taking into account the variability which is known to exist in part failure rates. Presently, reliability predictions are based on average part-failure-rate values and, thus, lead to predictions of system reliability which are, in turn, average reliability values. It is well established, however, that part failure rates observed in operating applications can differ significantly from average values. As a consequence, the true reliability of any one particular system may also differ from the reliability prediction of the average system.

In this report, methods of estimating failure-rate variability for the individual units of a system and of combining this information into system-level variability information are developed. If the variability inherent in the system-reliability prediction is known, it then becomes possible to state, as a probability, the chance that a particular system under development will meet or exceed a specified reliability goal.

The complement of this probability, the chance of failing to meet the reliability goal, is called the reliability risk or risk in this report. The risk probability is dependent on the information available at the time the computation is made. Early in a hardware development program, the information on part failure rates is not well known and the variability in the estimate would be large, therefore, the reliability risk will tend to be large. As the program progresses, more information becomes available, particularly through testing, and the variability inherent in the reliability estimate decreases. The reliability risk does not necessarily decrease as the program progresses but the precision of the reliability prediction is increased by the addition of more information. The reliability risk reflects how the current system stands relative to the reliability goal. By assessing the current information, the reliability analyst will know whether the system has a better or worse chance of meeting the reliability goal, that is, whether the system has a lower or higher reliability risk than at a previous point in the program. The reliability risk is a measure of the system reliability status at any point in the development program and incorporates into the estimate all the information available up to that time.

This report is specifically directed to the development of two techniques:

1. Risk determination, and
2. Resources allocation for reliability demonstration testing.

The allocation of resources for reliability demonstration testing is a direct consequence of being able to measure the uncertainty in the reliability predictions for the individual units and the system. The testing effort is allocated between units and system in such a way as to reduce the uncertainty in the reliability estimate of the system to a minimum for the resources expended in testing.

Substantial improvement can be expected in the planning of reliability tests by the use of failure-rate variability information. The conventional approach to planning suffers in two ways:

1. It does not use all available reliability information as, for example, past experience; it does not allocate effort differently in testing well known and little known units.

2. It is difficult to integrate the information gained from all program tests in a satisfactory manner to answer the question of the adequacy of system reliability.

The proposed approach to test-effort allocation makes use of information which is generally available in the development program and applies testing effort where it is most needed, i. e., on the least known units.

Present development programs must approach the assessment of product reliability subjectively. The objective phase, the reliability demonstration, is generally inconclusive statistically because of the limited resources available for test (dollars, test items, time). The final reliability decision ... is the reliability of the system adequate? ... will be based largely on engineering judgment. The reliability techniques proposed in this report are not less subjective; however, the subjective judgment is applied at a more appropriate level of the system.

Instead of asking for engineering judgment on overall system reliability, the engineering judgment is directed into specific areas concerning the adequacy of parts, design, assembly processes, and so on. These proposed techniques then integrate this information and testing information in a consistent manner up to the system level.

The techniques proposed herein are based on Bayesian statistics as opposed to conventional (Neyman-Pearson) statistics. Admittedly, the techniques proposed are not the only way to accomplish the goals but they are logically developed and do not violate any intuitive feelings about reliability or engineering.

Bayesian statistics is not new, but until recently has been held in disrepute by the statistical traditionalists. In the Bayes version of statistics, a parameter to be estimated is assumed to have a known *a priori* distribution. The traditionalists argue that in most cases this is a logical inconsistency; a parameter has but a single value and is not distributed over a range at all. This is true of many types of parameters such as the average height of all humans living today, for example. But in another sense, parameters do have distributions. For example, the failure rate of 5654 tube types, a parameter, does have a distribution if the total population of 5654 tube types as divided into separate lots by vendors is considered. The failure rates for individual lots are not identical; the lot failure rates could be represented by a statistical distribution. A second objection is, even if it is proper to speak of a distribution of a parameter, the exact form of the distribution will be unknown. It is acknowledged that an a priori distribution is partially subjective, but this is not more serious an error than the usual assumption of normality frequently made in conventional statistical practice.

The techniques developed here show the way to use more of the information available to the reliability analyst. Even so, it is conceded that the consequences implicit in these methods have been barely touched. Considerable benefit can accrue by an application of these and similar techniques in overall program planning, in the matter of optimum scheduling of the sequence of hardware development, and in the use for the evaluation of proposed program development plans.

THE DETERMINATION OF RELIABILITY RISKS

A NEW TECHNIQUE OF RISKS DETERMINATION

Present reliability analysis techniques furnish to program management an estimate of system reliability and it is partially on the basis of how close or far this estimate is to the reliability goal for the system that management judges the risks inherent in the development program. Obviously, other things are considered in assessing the risk of developing a reliable product--such questions as is the development pushing the state-of-the-art, are the component parts of the system well known from experience, have the manufacturing processes been tried successfully, has the operating environment been adequately described? The original system reliability prediction is modified little throughout the course of the development program unless major design changes are made, however, the judgment of the other factors can change the assessment of risk materially.

The present reliability analysis techniques predict the expected reliability of the system; that is, they furnish a single value which represents the best estimate of system reliability under normal or average conditions. This estimate of reliability ordinarily does not reflect any uncertainty in the input data used to make the prediction, nor does it ordinarily account for the manner of packaging the system or workmanship or process of assembly. The prediction by present methods must be viewed as an average or nominal value, and subsidiary information must be used to determine whether it is a good or a questionable estimate of the reliability of the system being developed.

The techniques proposed herein incorporates the element of uncertainty into the computation of risk. Instead of representing the reliability estimate as a single point, the estimate is represented by a range of values which is likely to contain the true reliability value. The range would be wide or narrow depending on how well the reliability of the system can be known. In this technique, the idea of range is carried a step further; the estimate of system reliability is represented in terms of a probability distribution whose mean value is equal to the best estimate of reliability by conventional techniques and whose variance is a measure of the uncertainty of this estimate. If such a distribution can be determined, then the reliability risk can be expressed as the probability that the system under development will not achieve the reliability goal. This statement of risk would have the advantage of including in it many of the elements of uncertainty which now are handled on a judgment basis in assessing development risk.

In order to derive a distribution for the reliability estimate, the possible causes of deviation from the estimate of the average must be analyzed and given numerical weights. Since this could be an exceedingly difficult task if considered at the system level, the system is first divided into smaller, physically separable hardware units for ease of analysis. A variance analysis and an estimate of mean failure rate is made on each of these units and then the results are combined in a statistical manner to arrive at the distribution of the estimate of system reliability. The subdivision of the system into separate hardware units is somewhat arbitrary though it is suggested two principles be observed: (1) the unit should be built under the cognizance of a single group (probably a single subcontractor) and (2) the unit should be an entity which is capable of being tested as an entity--that is, one whose performance can be evaluated independently of the performance of another unit of the system. In practice, it will probably prove most advantageous to keep the unit at the major component or assembly level but below the subsystem level.

The first problem to be considered is the analysis of uncertainty in the reliability predictions. The next section describes the development of a unit failure rate model which, in the estimate of variance considers factors of uncertainty due to the part types composing the unit, the circuit design, the packaging design, and the assembly processes. The following section explains the technique for combining unit reliability information into a distribution of the system reliability estimate and gives the formula for computing the a priori reliability risk. In the final section, a technique for incorporating test information into the estimates of reliability and reliability risk is developed.

FAILURE RATE VARIABILITY

1. Causes of Unit Failure Rate Variation

The true failure rates of the parts in a particular unit or system will most likely differ from the values used in the analysis. Even though certain deterministic adjustments are made to the basic part failure rate to reflect the particular stresses seen by the part in the system under consideration, there are still non-deterministic, random factors for which there is no adjustment. One part vendor consistently turn out a better product than another. If this vendor is selected, the true failure rate would be lower than the average failure rate; if the other vendor were selected, the true failure rate would be higher. The only way to achieve the average failure rate would be to use a mix of parts from all of vendors represented.

Sources of unknown, random variations in failure rates can be classified as follows:

1. Parts:

   Procurement: Vendor-to-vendor variation, lot-to-lot variation within vendor.
   Processing: Variation in incoming inspection, such as screening and aging.

2. System Design: Variation in circuit design tolerances to part parameter drift.

3. Packaging Design: Variation in protection from the physical environment.

4. Assembly Processing: Variations in workmanship, inspection, etc.

The effects from these sources can only be known statistically, i.e., as average values, and by some measure of variability, such as the variance or the standard deviation.

2. A Failure Rate Model

It is possible to construct a failure rate model which reflects both deterministic and random variations in failure rate values. A simple, yet realistic model for the part failure rate is

$$\lambda' = K\lambda,$$

where λ' is the true part failure rate value in the particular application, λ is the average part failure rate value including adjustments for derating, etc., and K is a further multiplicative adjustment factor.

The multiplicative factor K can be partitioned into independent, additive factors:

$$K = a + b + c + d,$$

where the factors a, b, c, and d correspond to adjustments in the average failure rate attributable to part procurement and processing, circuit design, packaging design, and assembly processing.

The expected value of the part failure rate in the particular system application is written as

$$\begin{aligned} E(\lambda') &= E\left\{K\lambda\right\} = \lambda E(K) \\ &= \lambda\left\{E(a) + E(b) + E(c) + E(d)\right\} . \end{aligned}$$

If the particular system is "typical" in circuit design, packaging design, assembly processing, and the selection of parts, then

$$E(K) = E(a) + E(b) + E(c) + E(d) = 1$$

and

$$E(\lambda') = \lambda .$$

In the "typical" system the expected values of a, b, c, and d have been determined by experience. The contribution of the different causes of failure in the average or typical equipment based upon published data is presented in the following table.

<u>The Contribution of the Different Causes of Failure in a Typical Equipment</u>

Cause of Equipment Failure	% of System Failure	Nominal Estimate of Factor
Unreliability of Parts	35	$E(a) = 0.35$
Circuit Design	35	$E(b) = 0.35$
Packaging Design	10	$E(c) = 0.10$
Assembly Processing	20	$E(d) = 0.20$
Total	100%	1.00

For the system using average parts, a value of $E(a) = 0.35$ would be assigned to reflect the contribution of part unreliability to all system failures. This is the <u>expected value</u> of <u>a</u> for a system using <u>average parts</u>.

If the system uses high reliability parts instead of average parts, there would be justification in lowering E(a) to, say, 0.25.

This adjustment of a does not change the expected values of b, c, and d, which remain 0.35, 0.10, and 0.20, respectively. The new sum of the expected values of a, b, c, and d is now 0.90.

This means that, because of the use of improved parts, obtained through processing and/or procurement, there would be only 90 percent as many failures in the system as would occur if average parts were used. The expected values of the factors b, c, and d would be adjusted from their values in an average system only when specific action is taken in the development program in each of the areas represented by these factors.

The expected value of part failure rate after adjustments is not necessarily the exact part failure rate. There are still unknown causes of failure rate variation, apart from the known causes accounted for in the adjustments of the expected values of the factors a, b, c, and d. For example, if the part failure rate is adjusted because high reliability parts are used, there can still be variations about the expected value because of differences between vendors supplying the parts. Similarly, the exact effect due to packaging design, and assembly processing would not be known. Only average or expected adjustments can be made.

The uncertainty element in the part failure rate is expressed as a variance about the expected value. The variance of part failure rate is

$$V(\lambda') = [V(a) + V(b) + V(c) + V(d)]\ \lambda^2 ,$$

where the terms V(a), V(b), . . . , are the variances of the individual factors.

The failure rate model can be extended to a higher level of the system by adding the expected part failure rates to obtain the expected failure rate of the higher order unit, and by adding part variances to obtain the variance of the unit. In the cases of part redundancy in the unit design, certain mathematical adjustments must be made in the summation. Care must also be exercised in applying the failure rate factors a, b, c and d since they operate at different levels of the unit.

The a factor operates at the individual part level, i.e., it is determined by vendor selection and incoming part processing. The b factor operates at the circuit level and is determined by circuit design. The c factor operates at the assembly level and is determined by the packaging design. The d factor operates on the unit level and is determined by the assembly processing.

Each factor has an effect upon a part so it is convenient to express the part failure rate model as

$$\lambda'_{ijk\ell} = (a_i + b_j + c_k + d)\, \lambda_{ijk\ell},$$

where

$i = 1, 2, \ldots, r,$ the number of part types

$j = 1, 2, \ldots, s,$ the number of circuits

$k = 1, 2, \ldots, t,$ the number of assemblies

$\ell = 1, 2, \ldots, n_{ijk},$ the number of parts of the i^{th} type in the j^{th} circuit and the k^{th} assembly.

$\lambda_{ijk\ell}$ is the tabulated failure rate for the part including adjustments for derating, etc.

a_i is the part type effect.

b_j is the circuit design effect.

c_k is the assembly package design effect.

d is the assembly processing effect.

$$\sum_{\ell} \lambda_{ijk\ell} = n_{ijk}\, \lambda_{ijk} \text{ since } \lambda_{ijkn} = \lambda_{ijkp}.$$

The unit failure rate is given by

$$\lambda'_u = \sum_i \sum_j \sum_k \sum_{\ell} (a_i + b_j + c_k + d)\lambda_{ijk\ell}$$

$$= \sum_i \sum_j \sum_k n_{ijk} (a_i + b_j + c_k + d)\lambda_{ijk}$$

$$= \sum_i a_i \left(\sum_j \sum_k n_{ijk} \lambda_{ijk} \right) + \sum_j b_j \left(\sum_j \sum_k n_{ijk} \lambda_{ijk} \right)$$

$$+ \sum_k c_k \left(\sum_i \sum_j n_{ijk} \lambda_{ijk} \right) + d \sum \sum \sum n_{ijk} \lambda_{ijk}$$

$$= \sum_i a_i \; T_{i..} + \sum_j b_j \; T_{.j.} + \sum_k c_k \; T_{..k} + dT_{...} ,$$

where $$\sum_j \sum_k n_{ijk} \lambda_{ijk} = T_{i..} ,$$

$$\sum_i \sum_k n_{ijk} \lambda_{ijk} = T_{.j.} ,$$

$$\sum_i \sum_j n_{ijk} \lambda_{ijk} = T_{..k} ,$$

$$\sum_i \sum_j \sum_k n_{ijk} \lambda_{ijk} = \lambda_{...} = T_{...}$$

The λ's are fixed and known, while the effects vary. Therefore, we have

$$E(\lambda'_u) = \sum_i E(a_i)\, T_{i..} + \sum_j E(b_j)\, T_{.j.} + \sum_k E(c_k)\, T_{..k} + E(d)\, T_{...}$$

and

$$V(\lambda'_u) = \sum_i V(a_i)\, T^2_{i..} + \sum_j V(b_j)\, T^2_{.j.} + \sum_k V(c_k)\, T^2_{..k} + V(d)\, T^2_{...}$$

The T's are easily computed since they are merely the sums of tabulated failure rates for each classification, i. e., part types, circuit, and assembly.

The computation of the unit expected failure rate and variance are most easily carried out in tabular form. The estimates of the mean and variance of the factors E(a), V(a), E(b), V(b), ... etc., are determined by the method shown in Appendix B. Once these values have been determined for each part type, circuit, and assembly, they are combined with the standard, adjusted part failure rates as shown in the following tables.

Parts by Type	Quality	Adj. FR per part	Total FR	E(a) x Total	V(a) x Total
Resistor, Type A	10	0.001×10^{-6}	0.01×10^{-6}	$(0.30)(0.01 \times 10^{-6})$	
Resistor, Type B	5	0.005×10^{-6}	0.025×10^{-6}	$(0.40)(0.025 \times 10^{-6})$	
Capacitor, Type C	3	0.003×10^{-6}	0.009×10^{-6}	$(0.35)(0.009 \times 10^{-6})$	
. . .					
Unit Total			T . . .	E(d) T . . .	$V(d)\ T^2$ . . .

Circuits/black box	CCT FR	E(b) x Total	$V(b)\ (Total)^2$
Amplifier #1			
Amplifier #2			
. . .			
Black Box #1	CCT FR	E(b) x Total	$V(b)\ (Total)^2$

Flip-Flop #3			
Oscillator #1			
. . .			
Black Box #2	CCT FR	E(b) x Total	$V(b)\ (Total)^2$

INTEGRATION OF RELIABILITY INFORMATION

1. A Priori Distribution

If the units of the system are in series in the reliability sense, then the expected value of the system failure rate is the sum of the expected values of the unit failure rates. The variance of the system failure rate estimate is similarly, the sum of the unit variances. It is useful to represent the information known about the system's expected failure rate and variance in terms of a probability distribution. Such a distribution would be called an *a priori* distribution, since it is derived from information existing at the moment and prior to any additional information obtained from testing.

An *a priori* distribution can be interpreted in two ways.

Since the failure rate of the system is unknown and could theoretically be anywhere within the range of zero to infinity, the probability distribution is a measure of one's judgment that the failure rate may be at some specific value. This interpretation of the *a priori* distribution--a measure of one's best judgment--is used later in conjunction with life test results.

Another interpretation is that the a priori distribution represents a real or hypothetically realizable population of failure rates. It represents the population of failure rates of all systems of the same generic type which could be built, using parts from all vendors, all workmen, and so on. Since there are variations in parts, workmanship, packaging design, etc., the failure rates of the individual systems will vary for the population and would be represented by a probability distribution.

If the a priori distribution is to represent a population of system failure rates, the nature of the population must be examined in light of the type of development program being considered. Spacecraft development is representative of small scale development and production programs. It is safe to assume in such programs that the limited number of systems to be built will be built to the same design, will be assembled by the same workmen using the same assembly processes, and the parts will come from the same vendor lot. The individual systems are nearly identical and the failure rates of these systems will be nearly identical.

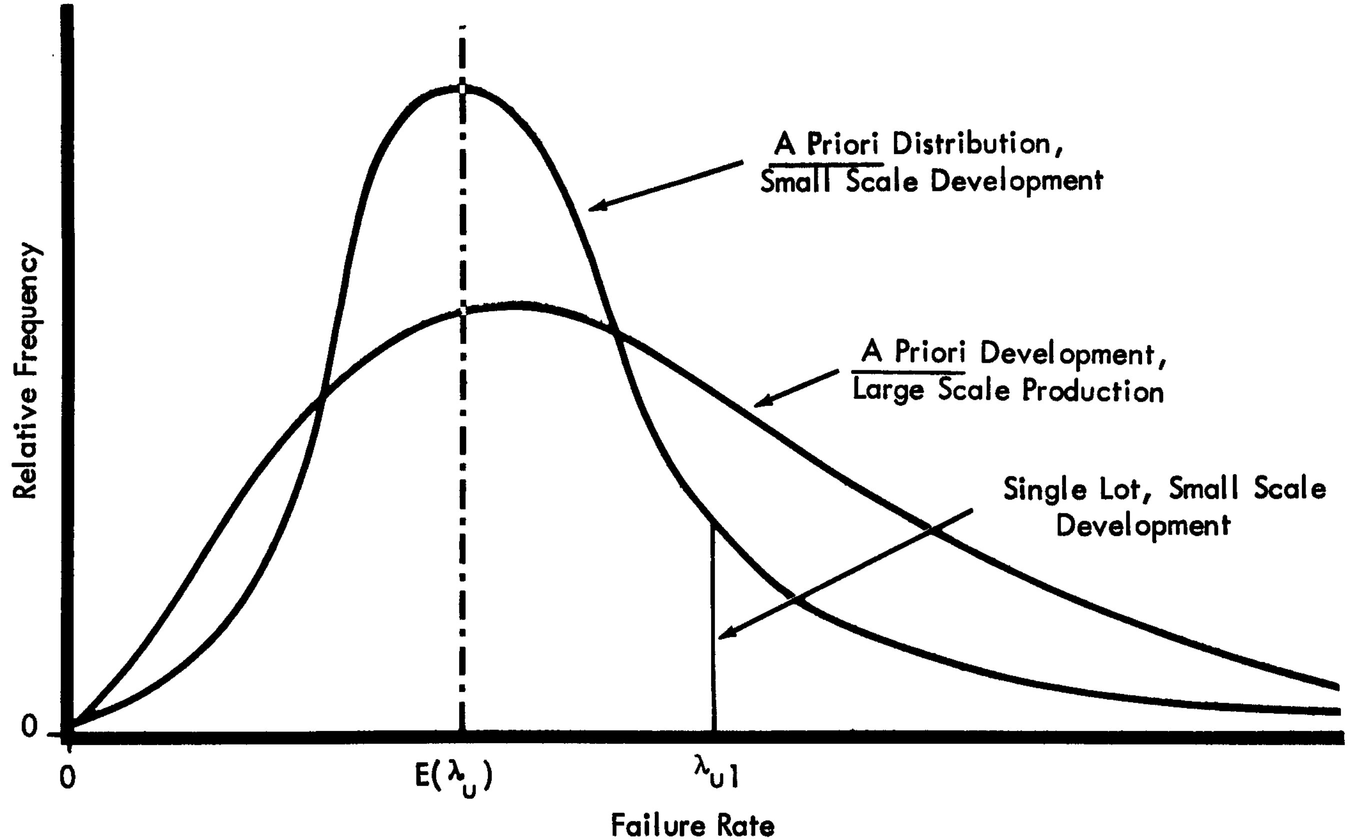
If the systems are produced by large scale production programs with less control, there will be more variation between the systems. Even within the same large scale production lots, there can be variation between system failure rates if, for example, parts of the same type used in the systems come from different part vendors.

Figure 2 shows two a priori distributions, one applicable to large scale production and the other to small scale production. The failure rate for systems from a single small scale production program is represented as λ_{u1}.

The failure rate for systems from a single large scale production lot cannot be represented by a single value, since the individual systems from the same production may have different failure rate values. Prior to any test results, the best estimate of system failure rate for large scale production is $E(\lambda_u)$, the same as the average failure rate for small scale production systems.

The major distinction between large and small scale production is in the magnitude of the variation between system failure rates. In large scale production, less control is exercised and a larger variation between systems results. In both cases the average system failure rate is the same. The inferences to be

FIGURE 2
Distributions of Unit Failure Rate for Different Types of Production



drawn from life testing will also differ between large scale and small scale production. Life test results from a sample of systems from a single small scale production will lead to valid inferences about the remaining systems in the lot because of the homogenity of the systems. Life test results on systems from large scale production lead to less valid inferences about the remaining systems because of the greater heterogeneity between systems.

2. The A Priori Probability for Reliability Risk

Many of the units to be procured for a spacecraft development program will come from small scale production lots and the systems can be considered a product of small scale production. While the failure rate of these systems cannot be known it is possible to compute a probability the system will not meet the specified reliability goal. This probability is the reliability risk and is dependent on the a priori distribution of system failure rate.

Assume that a critical value of system failure rate (λ_c) can be determined to correspond to the reliability goal. If the true failure rate of the system being built is less than λ_c, the system will be adequate; otherwise it will not be. If the a priori distribution of system failure rate is $g(\lambda)$, then the reliability risk is defined as

$$P\left\{\lambda > \lambda'_c\right\} = \int_{\lambda=\lambda_c}^{\lambda=\infty} g(\lambda)\, d\lambda .$$

In order to determine this probability, we need to know the functional form of $g(\lambda)$. From practical considerations, the failure rates must always be positive and limited failure rate data suggest the distribution ought to be skewed. Several statistical distributions meet the requirements of skewness and positive range on the variable. Among these are the log normal, the Weibull, and the gamma. The normal distribution could be used as an approximation providing $\mu >> \sigma$; however, skewness is zero.

The most reasonable choice for the a priori distribution of system failure rates appears to be the gamma distribution. The gamma probability distribution leads to tractable mathematical analysis; it is a flexible distribution which can fit a variety of empirical distributions quite well and it gives a good fit to the limited empirical data on failure rates.

The gamma is a two-parameter distribution which can be written in the form

$$g(\lambda) = \frac{\lambda^{n-1} \quad e^{-\lambda/\beta}}{(n-1)! \quad \beta^n} \qquad \lambda > 0, \quad \beta > 0, \quad n \geq 1$$

where β is a scale parameter and n is a shape parameter. If $n = 1$, the gamma reduces to the well known exponential distribution. For $n > 1$, the gamma is unimodal. For n sufficiently large, the gamma approximates the normal distribution.

The mean and variance of the gamma distributions are, respectively,

$$E(\lambda) = \beta n$$

$$V(\lambda) = \beta^2 n.$$

Using the estimated mean and variance for system failure rate, the parameters of the <u>a priori</u> distribution can be determined as

$$\beta = V(\lambda)/E(\lambda)$$

$$n = E^2(\lambda)/V(\lambda).$$

By substituting the numerical values of the parameter into the *a priori* distribution, the reliability risk can be computed by the integration indicated earlier. However, since the chi-square distribution (a special case of the gamma for $\beta = 2$ and $n = V/2$) has been extensively tabulated by fractiles of the distribution, it is useful to transform the *a priori* distribution into a chi-square. By a simple transformation, the reliability risk for the system becomes

$$P\left\{\lambda > \lambda_c\right\} = P\left\{\chi^2_{2n} > \frac{2\lambda_c}{\beta}\right\},$$

where χ^2_{2n} is a chi-square variate with 2n degrees of freedom $\left\{n = E^2(\lambda)/V(\lambda) \text{ and } \beta = V(\lambda)/E(\lambda)\right\}$. This probability can be easily determined from tables of the chi-square distribution.

It should be emphasized that the reliability risk is a probability statement based on the evidence available at the time the probability is computed. It is, so to speak, the odds against a successful reliability development. As more information is accumulated, particularly from life testing, the odds should change.

The approach used to determine the _a priori_ reliability risk for the system is a Bayesian approach. It is characterized by the assumption that the distribution of a parameter (the failure rate) exists and is known. It differs from the more conventional Neyman-Pearson approach to statistical inference which is based only on sample evidence and not on _a priori_ evidence. In Neyman-Pearson statistics, there can be no probability statement comparable to the _a priori_ probability of development success.

If it is desired, the reliability risk for any unit of the system can be determined in the same manner as for the system. Assuming the gamma distribution is an appropriate _a priori_ distribution of unit failure rate, the estimates of the parameters β and n would be made from the estimates of unit failure rate and variance. It is then possible to compute the unit reliability risk relative to a critical value λ'_c.

UTILIZATION OF TEST INFORMATION

1. Best Estimate of Failure Rate after Life Testing

In Bayesian statistics, the results of life testing can be used to modify the _a priori_ estimate of system or unit failure rate. This is done by pooling the _a priori_ and the test information.

The *a priori* estimate of failure rate based on the preceding is

$$E(\lambda) = \beta n$$

where $E(\lambda)$ is the expected failure rate, β is the scale parameter, and n is the shape parameter.

If, as a result of life testing for T hours, r failures are observed, the best estimate, in the Bayesian sense, of failure rate becomes

$$E(\lambda \mid T, r) = \left(\frac{\beta}{1 + T\beta}\right)(n + r).$$

(The derivation of this expression is given in Appendix I.)

This is a function of the parameters of the *a priori* distribution and the results of life test: r failures in time T. This result is analogous to the previous expression where $\beta/(1 + T\beta)$ replaces β and $(n + r)$ replaces n.

2. An Uncertainty Distribution of Failure Rate

With the accumulation of life test results, the estimate of the failure rate and the variance of the estimate will change. The *a priori* distribution of failure rate no longer adequately represents all the available knowledge and it should be modified to accommodate the addition of test information.

However, if the a priori distribution is modified, it can no longer represent a population as originally conceived, since a population must be fixed and invariant with time. In order to distinguish between the original a priori distribution and the modified, the latter is called an uncertainty distribution. It is analogous to a sampling distribution in conventional statistics except that it incorporates a priori information on the distribution of the parameters.

The uncertainty distribution is of the same mathematical form as the a priori distribution (a gamma probability density function), but the expectation and variance will differ as a result of the additional test information.

The importance of the uncertainty distribution lies in its use in computing a new estimate of the reliability risk. Since the uncertainty distribution is a gamma probability density, the reliability risk after testing can be written as

$$P\left\{\lambda > \lambda_c\right\} = P\left\{\chi^2_{2(n+r)} > 2\lambda_c\left(\frac{1+T\beta}{\beta}\right)\right\}$$

This is similar to the *a priori* risk expression (see below) with two exceptions: the degrees of freedom for the chi-square becomes $2(n + r)$, and the original $1/\beta$ term now becomes $(1 + T\beta)/\beta$. Then n and β are the parameters of the original *a priori* distribution. The r and T are the test results (r failures in time T).

In practice, it is not necessary to compute the parameters n and β of the *a priori* distribution used in the risk equation since they can be easily expressed in terms of the mean and variance of the failure rate estimate. The previous expression for the reliability risk probability can be written as

$$P\left\{\lambda > \lambda_c\right\} = P\left\{\chi^2_{2n'} > 2\lambda_c/\beta'\right\}$$

where

$$n' = E^2(\lambda')/V(\lambda')$$

$$\beta' = V(\lambda')/E(\lambda')$$

and $E(\lambda')$, $V(\lambda')$ are the estimates of failure rate mean and variance after testing.

If testing is at the unit level, the estimate of mean and variance of unit failure rate determined after testing can be expressed in terms of the estimates made before testing and the test results. These formulas are:

$$E(\lambda') = E(\lambda|r,\ T) = \left[E^2(\lambda) + rV(\lambda)\right] / \left[E(\lambda) + TV(\lambda)\right],$$

$$V(\lambda') = V(\lambda|r,\ T) = \left[E^2(\lambda) + rV(\lambda)\right] V(\lambda) / \left[E(\lambda) + TV(\lambda)\right]^2 .$$

To determine the appropriate estimate of mean and variance of the system failure rate as a consequence of unit testing, add the new estimates of unit means to obtain the system mean and add the new estimates of unit variances to obtain the system variance. These estimates of the system mean and variance would then be used to determine n' and β' and, thus, the reliability risk for the system.

If in addition to unit testing, the system also is tested as an entity, the information from system testing is incorporated into the estimate of system mean and variance by the above formulas. The T and r values are the results of the system test and the $E(\lambda)$, $V(\lambda)$ would be the estimates of system mean and variance after unit testing. The estimates $E(\lambda|T,\ r)$, $V(\lambda|T, r)$

would then be system estimates based on both unit and system testing and these would be used to establish n' and β' and, in turn, establish the reliability risk for the system.

RESOURCES ALLOCATION FOR RELIABILITY TESTING

RELIABILITY DEMONSTRATION TESTING

There are three types of tests used in a development program: development tests, qualification tests, and acceptance tests. These tests are usually performed in time sequence, but there may be exceptions depending on the level of the system being tested. All these tests furnish some reliability information about the product under development, but all of the information is not equally usable in the type of reliability analysis to be considered now.

In the technique for allocating resources for reliability demonstration, interest is in those tests which furnish information which can be used to estimate the achieved reliability of the system. This information is statistical in nature and will be generated by life tests having as the observed variable the number of failures occurring during a fixed time or fixed number of operating cycles. Tests of this sort--generally life tests--will be considered reliability demonstration tests in this report.

THE INADEQUACY OF THE CONVENTIONAL RELIABILITY DEMONSTRATION

More and more development programs require some reliability demonstration, but these demonstrations are generally inconclusive by conventional statistical standards. The difficulty lies in the excessive cost of testing and the length of test time required for statistical proof.

For example, suppose the reliability requirement for a spacecraft system is a 70 percent chance of survival for one year. A conventional statistical criterion of reliability demonstration might be to test the system for 1.95 years, without failure, in order to have a 50 percent certainty that the requirement has been met. It is obvious this type of test requirement is beyond the scope of most spacecraft development programs.

The burden then falls on the program manager to weigh the relative worth of whatever life test data he can get and then use his own judgment in arriving at the decision to accept or reject the system because of its reliability.

The conventional statistical design of a reliability demonstration test ignores any previous information concerning the reliability of the product to be tested. The same statistical criterion would be used if the product was known to have a low reliability or a high reliability since the criterion is based only on the reliability goal for the product and on the sampling risks one is willing to assume.

AN OPTIMUM ALLOCATION TECHNIQUE

Because of the inadequacy of the usual reliability demonstration, a need for a new approach is evident. The conventional statistical design of a reliability demonstration is in terms of the number of test units and the length of testing needed to prove statistically the achieved reliability of the product. This may be unrealistic in that the quantity of units and time may be far beyond the scope of the program. A more realistic approach to the problem is to develop a test program which furnishes the maximum system reliability information for the resources available for testing. This report is concerned with the development of such a technique.

The first problem to be considered is to establish an appropriate measure of system reliability information. Next an information equation relating the test times for units to an equivalent test time for the system is developed. By use of the information equation in conjunction with program cost information and schedule information it is possible by the solution of a simple linear programming problem, to arrive at an optimum allocation of resources to reliability demonstration testing.

The technique which is developed can be applied repeatedly at successive stages of the development program. The allocation made at any step of the development program is always based on the expected results in the next phase of testing. Either this expectation is met or it is not, so that before beginning the next phase of testing the remaining resources may have to be reallocated to keep the test program at an optimum.

1. <u>A Measure of Reliability Information</u>

One obvious measure of reliability information is in the failure rates for both units and system. This would be an appealing measure since it is a simple matter to relate the information known about the units to the information known about the system; the system failure rate being the sum of unit failure rates in the series case. This measure of reliability information is not sufficient as a basis for allocating future test effort. The estimates of system and unit failure rates are not expected to chance under future testing and, therefore, there can be no basis for a preference in testing of one unit over another or over the system. If one expects a change in a failure rate value as a result of testing, then better information about the failure rate is available and should replace the original estimate.

By virtue of the Bayesian analysis used in the last chapter to establish the a priori and uncertainty distribution for failure rate, it is possible to use the variance of the failure rate estimate as a measure of reliability information. The variance is easy to relate to unit and system information since the system variance is the sum of unit variances. The variance, as a measure of the quality of the failure rate estimate, will change under testing. Variance, or its reciprocal, is commonly used as a measure of information where the smaller the variance, the better the estimate.

Unfortunately, there are drawbacks in the use of variance when the a priori and uncertainty distributions are gamma probability density functions. The gamma function is not like the normal, in that the mean and variance of the distribution are functionally dependent. For the gamma function, the relation between mean and variance is,

$$V(\lambda) = \beta E(\lambda) = E^2(\lambda)/n$$

where β and n are the function parameters. For a fixed β or a fixed value of n, the larger the mean, $E(\lambda)$, the larger the variance, $V(\lambda)$.

This dependency between the mean and variance suggests another, more appropriate measure of reliability information, namely the coefficient of variation which is defined as

$$CV = \sqrt{V(\lambda)/E^2(\lambda)} .$$

For the gamma function the reciprocal of the coefficient of variation squared is the parameter n, the shaping parameter. The CV is independent of β, the scale parameter which is to say the CV is not dependent on the location of the mean value. The parameter n can be related by analogy to sample size. The larger the sample size the better known is the sample estimate and is independent of whether the sample is estimating a large value or a small value. Similarly, the larger the value of n, the smaller the variance relative to the mean, the more precisely the mean can be located independently of whether the mean is a large or small value.

Because of the advantage of being independent of the mean value, the CV (CV = $n^{-1/2}$) has been chosen as the appropriate measure of reliability information. It is not as simple to relate unit and system reliability information through the system CV as it is the variance. If λ_s stands for the system failure rate estimate and λ_i for the failure rate estimate of the i^{th} unit, then the square of the system CV, in terms of unit values is,

$$(CV)^2 = V(\lambda_s)/E^2(\lambda_s) = \sum V(\lambda_i)/\sum E^2(\lambda_i).$$

2. Information Equation

To allocate testing to the various units of the system and/or to the system, within the resources available in the program, requires an ability to estimate the gain in information from various combinations of units and system testing. For this, an information equivalence relation is developed. We shall say that testing the i^{th} unit of the system for x_i hours is equivalent to testing the j^{th} unit x_j hours if the expected decrease in the system's CV is the same in both cases. If the i^{th} unit is tested, the system CV will become,

$$CV = \frac{\left[V(\lambda_s) - V(\lambda_i) + \left(\frac{\beta_i}{1 + x_i \beta_i} \right)^2 (n_i + r_i) \right]^{1/2}}{E(\lambda_s) - E(\lambda_i) + \left(\frac{\beta_i}{1 + x_i \beta_i} \right) (n_i + r_i)} .$$

The last terms in both the numerator and denominator are the expressions for variance and mean respectively, of the i^{th} unit after testing x_i hours. (See Appendix A). Before the test starts, the best estimate of λ_i is $\beta_i n_i$ and, therefore, the expected number of failures, r_i, is $\beta_i n_i x_i$. Substituting this expected value of r_i into the expression, the system CV becomes

$$CV = \frac{\left[V(\lambda_s) - V(\lambda_i) + \frac{\beta_i^2 n_i}{(1 + x_i \beta_i)} \right]^{1/2}}{E(\lambda_s) - E(\lambda_i) + \beta_i n_i} .$$

Testing of the j^{th} unit for x_j hours leads to a similar expression with the subscripts i replaced by j. If these two expressions of system CV are equated and simplified, then

$$x_i = \frac{\beta_j V(\lambda_j)\, x_j}{\beta_i V(\lambda_i) - \beta_i \beta_j [V(\lambda_j) - V(\lambda_i)]\, x_j} .$$

This is an equality of information equation in that it determines the amount of testing x_i for a specified x_j so that the change in system CV is equal.

Using the same argument it is possible to establish the equality of information equation between unit testing and system testing. If the system is tested for x_s hours, the system CV will become

$$CV = \frac{\left[\left(\frac{\beta_s}{1 + x_s \beta_s}\right)^2 (n_s + r_s)\right]^{1/2}}{\frac{\beta_s}{1 + x_s \beta_s} (n_s + r_s)} .$$

Substituting the expected value $\beta_s n_s x_s$ for r_s, this becomes

$$CV = \frac{\left[\frac{\beta_s^2 n_s}{1 + x_s \beta_s}\right]^{1/2}}{\beta_s n_s} .$$

Equating this expression to the expression for the i^{th} unit and simplifying, x_s becomes

$$x_s = \frac{\beta_i V(\lambda_i) x_i}{\beta_s V(\lambda_s) - \beta_i \beta_s [V(\lambda_i) - V(\lambda_s)] x_i} .$$

In most development programs the term in the denominator,

$$\beta_i \beta_s [V(\lambda_s) - V(\lambda_i)] x_i$$

will be negligible relative to $\beta_s V(\lambda_s)$, and this is true of the similar term in the equation for the i^{th} and j^{th} unit*. As a good approximation, therefore, these equality of information equations can be written as

$$x_s \doteq \frac{\beta_i V(\lambda_i)\, x_i}{\beta_s V(\lambda_s)} \quad ,$$

$$x_i \doteq \frac{\beta_j V(\lambda_j)\, x_j}{\beta_i V(\lambda_i)} \quad .$$

An information equation can now be derived from these approximations. Suppose the system is tested y_s hours, the i^{th} unit y_i hours, the j^{th} unit y_j hours and so on, then the total time in testing expressed in equivalent system hours is

$$y = y_s + \frac{\beta_i V(\lambda_i)}{\beta_s V(\lambda_s)}\, y_i + \frac{\beta_j V(\lambda_j)}{\beta_s V(\lambda_s)}\, y_j + \ldots \quad .$$

This is the information equation and it can be used to derive an optimum test program plan. The value of y can be used to calculate the expected reliability risks for the system after testing; the y would replace the T in the risk expression and $\beta_s n_s y$ would be used for r (see page 48).

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* See Appendix A for the case where this item is not negligible.

3. Allocation of Testing Effort

The resources available in a program which can be applied to reliability demonstration testing are: the total money available for testing, the number of items (units and systems) available for testing, and the elapsed time within the program schedule which could be used for testing.

The cost of testing will include the cost of procuring items for test, set-up costs, monitoring and repair costs, power costs, the cost of facilities, and the like. If the total cost of testing m_i items of the i^{th} unit for t_i hours is designated by $C(m_i, t_i)$, this cost can be set equal to the fixed costs, c_i, the initial cost of the items tested $m_i c_i$, and the time dependent costs such as monitoring and repair, $m_i k_i t_i$. Thus,

$$C(m_i, t_i) = c_i + m_i d_i + m_i k_i t_i.$$

In the typical development program there will be considerable restrictions on both m_i, the number of items available for test, and t_i, the time allowed for testing. The testing of the unit cannot start until it is developed and built, and the testing must finish sometime before the scheduled completion of the program. Any allocation procedure for which t_i extends beyond the end of the development program would be useless. Similarly an allocation procedure for which t_i is very short would be wasteful. Rather than allow t_i to be established by the allocation procedure, it is more realistic to fix the value of t_i at a value t_i', during program planning, in a manner which is consistent with the program schedule.

Reasonable values of m_i are likewise limited, though there should be some freedom in their choice. During program planning an upper bound on m_i, say M_i should be established in light of the development schedule and item cost. Prior to final allocation, the number of items of the i^{th} type to be tested can be considered a limited variable which may assume values 0, 1, 2, . . . , M_i.

By considering the test time per item to be fixed at t_i' and considering the items to be tested as a variable, m_i, the total cost of testing the i^{th} type units can be written as a function of the number of items tested,

$$C(m_i \mid t_i') = \delta_i c_i + K_i m_i$$

$$m_i = 0, 1, 2, \ldots, M_i,$$

$$\delta_i = \begin{cases} 0 \text{ if } m_i = 0 \\ 1 \text{ if } m_i \neq 0 \end{cases} .$$

The total program costs for testing, C_o, will be the sum of all unit test costs and system test costs,

$$C(m_i \mid t_i') = \sum_{\text{all units and system}} \left\{ \delta_i c_i + K_i m_i \right\} .$$

Since t_i' is fixed and m_i is a variable, the information equation can be written as a function of the m_i's. Let $y_i = m_i t_i'$, then

$$y = m_s t_s' + \sum \left[\frac{\beta_i V(\lambda_i)}{\beta_s V(\lambda_s)} \, t_i' \right] m_i$$

where m_s and t_s' are respectively the number of systems tested and the test time allowed for the system.

Both the total program cost and the information equation are linear functions of the m_i's. The total program money available for reliability demonstration testing will be limited to not more than some amount, C_o. It is now possible to find a set of values for the m_i's, which will maximize the information from testing, y, subject to the condition $C \leq C_o$. A step-wise procedure to solve for the m_i's (which can be easily programmed for a computer), is shown in Appendix B.

In order to gain insight into the allocation procedure, a graphical solution to a simple problem is shown here. A graphical solution would become unwieldy in a complex development program and it is not recommended as a means of solution. The graphical solution, however, does illustrate some pertinent points.

The solution is begun by constructing a table similar to the following for each testable unit of the system and for the system. The table illustrated is for the i^{th} type units which can be tested a maximum time, t_i', per unit and up to N_i units can be tested.

Information/Cost Table

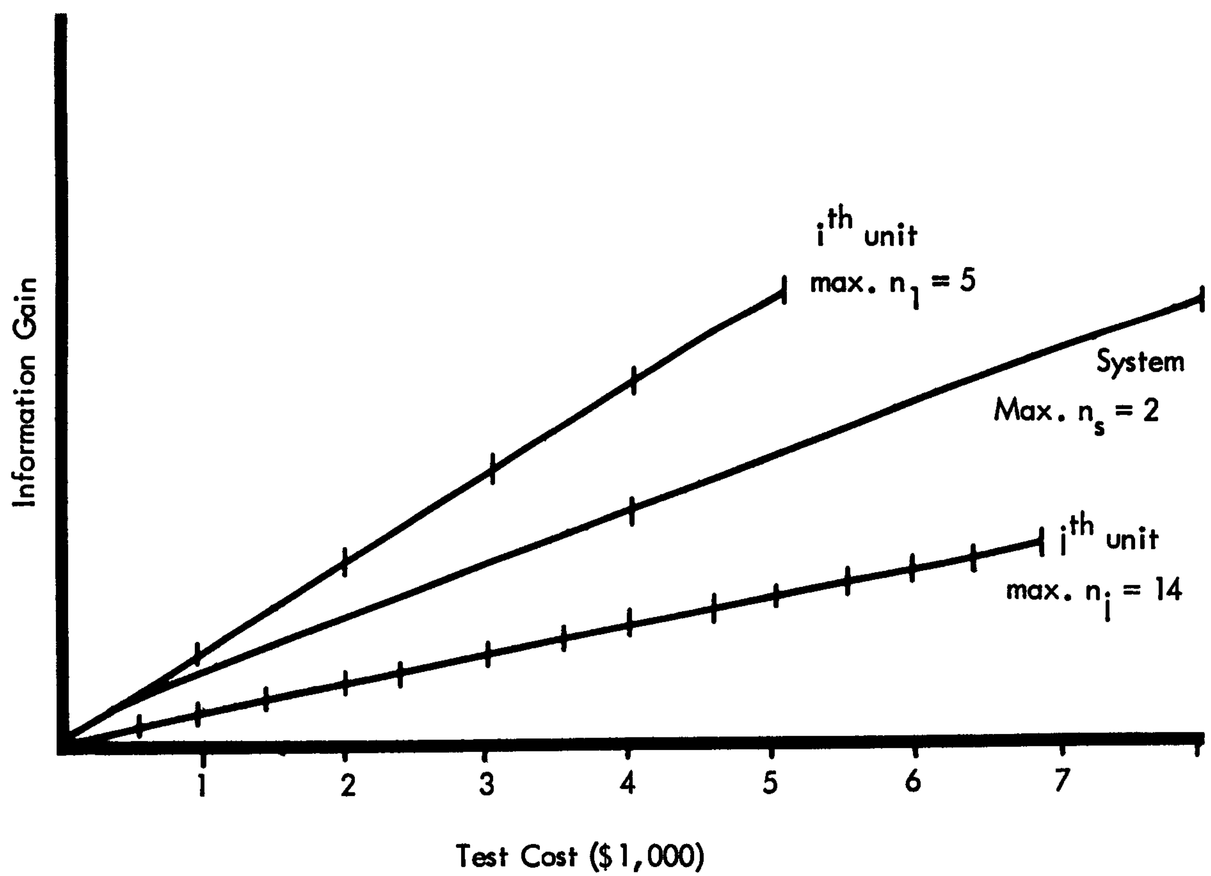
m_i	$C(m_i)$	Information Gain; Equivalent System Hours
1	\$1,000	$Y_i = t_i \beta_i V(\lambda_i) / \beta_s V(\lambda_s)$
2	2,000	$2Y_i$
3	3,000	$3Y_i$
4	4,000	$4Y_i$
5	5,000	$5Y_i$

The information gained by testing, $Y(m_i)$, can be graphed as a function of test costs, $C(m_i)$. This curve is plotted in Figure 3 along with similar curves for the other testable units and the system. The curves are represented as continuous lines though they really exist only at the points marked on the lines.

Suppose there is a total of C_o = \$4, 750 to spend on testing. From Figure 3 it is evident there are several ways to spend C_o. We might test either one system (\$4, 000), or four units of the i^{th} type (\$4, 500). None of these will exactly expend the \$4, 750 since partial testing is not allowed. If there is enough for four, but not for five unit tests, only four unit tests would be considered.

Of the three possibilities, the most information is gained from testing four units of the i^{th} type. This costs \$4, 000, leaving an excess of \$750 to be used for additional testing. With the remaining money, the best that can be done is to test one unit of the j^{th} type for \$500, thus, there is an excess of \$250 which cannot be spent.

FIGURE 3
Information Gain as a Function of Test Costs



The optimum allocation of testing effort within a limit of \$4,750 is to test four units of the i^{th} type for t_i hours each, and one unit of the j^{th} type for t_j hours. No other combination of testing can "buy" any more information for the same amount of money.

In the example shown, the information-cost relations were assumed linear though this need not be the case. It is reasonable to assume that the cost per unit test will decrease as the number of units tested increases up to the point where new test facilities must be built to accommodate one more unit. The graphical as well as the analytical solution can easily take into account nonlinearity in costs.

The graphical solution just shown suggests that this technique can be extended to become a useful tool in total program planning and in the assessment of a program plan. In determining the program money to be spent on testing or in evaluating the proposed test expenditure, it would be useful to know how much reliability information can be bought by testing. The problem just illustrated could have been solved for various values of C_o. The information gain for each C_o could then be evaluated to determine the appropriate amount of money to be allocated to total program testing.

APPENDIX A

MATHEMATICAL DERIVATIONS

APPENDIX A

MATHEMATICAL DERIVATIONS

DERIVATION OF BEST ESTIMATE

Three functions are defined. A decision statistic, which is some unknown function of the test results (r, T), is defined as $d(r, T) = d$. A loss function, $L(d, \lambda)$, is defined as

$$L(d, \lambda) = [d-\lambda]^2 .$$

The nature of the loss function is such that, when $d = \lambda$, $L = 0$; there are no losses when d exactly estimates λ, the unit failure rate.

The risk function, $R(d, \lambda)$, is the loss function averaged over all possible life test results ($r = 0, 1, 2, \ldots$) for fixed T:

$$R(d, \lambda) = E_r\left[\left\{L(d, \lambda)\right\}\right].$$

Assuming the unit has a constant failure rate (λ), the probability of exactly r failures in time T is

$$f(r \mid \lambda) = \frac{(\lambda T)^r e^{-\lambda T}}{r!}, \quad r = 0, 1, 2, \ldots .$$

The risk function then becomes

$$R(d, \lambda) = \sum_{r=0}^{\infty} (d-\lambda)^2 \left[\frac{(\lambda T)^r e^{-\lambda T}}{r!}\right] .$$

Since the a priori distribution of unit failure rate is a gamma function with parameters β and n, then the Bayes risk can be stated as the risk function averaged over all values of λ. If the a priori distribution of λ is designated by $g(\lambda)$, then the Bayes risk of d, relative to g, becomes

$$R(d|g) = \int_{\lambda} R(d, \lambda)\, g(\lambda) d\lambda$$

$$= \int_{o}^{\infty} \sum_{r=o}^{\infty} (d-\lambda)^2 \left[\frac{(\lambda T)^r e^{-\lambda T}}{r!}\right] \left[\frac{\lambda^{n-1} e^{-\lambda/\beta}}{(n-1)!\ \beta^n}\right] d\lambda.$$

After integration and some simplification, the Bayes risk can be rewritten as:

$$R(d|g) = \sum_{r=o}^{\infty} \left[\left\{d - \frac{\beta}{1+T\beta}(n+r)\right\}^2 + \left(\frac{\beta}{1+T\beta}\right)^2 (n+r)\right]$$

$$\left[\frac{(n+r-1)!}{r!\ (n-1)!}\right] \left[\frac{T^r}{\beta^n \left(T+\frac{1}{\beta}\right)^{n+r}}\right].$$

This risk will be minimized with respect to d if for all r, the term immediately following the summation sign above is zero. Thus the risk is minimized for

$$d = \frac{\beta}{1+T\beta}(n+r).$$

This value of d is the best estimate of λ in the sense of minimizing the Bayes risk. It is also a consistent estimate of λ because, as the test time (T) approaches infinity, the number of failures (r) approaches infinity, and the ratio (r/T) approaches λ. Thus,

$$\lim_{T\to\infty} d = \lim_{T\to\infty} \frac{\beta}{1+T\beta}(n+r)$$

$$= \lim_{T\to\infty} \frac{\beta n}{1+T\beta} + \lim_{T\to\infty} \frac{\beta}{\frac{1}{r} + \beta\left(\frac{T}{r}\right)}$$

$$= \lambda.$$

Using this value of d as the best estimate of λ, the evaluation of the Bayes risk becomes

$$R\left[\frac{\beta}{1+T\beta}(n+r) \mid g\right] = \left(\frac{\beta}{1+T\beta}\right)^2 \sum_{r=0}^{\infty} \frac{(n+r)!}{r!\,(n-1)!} \frac{(T\beta)^2}{(1+T\beta)^{n+r}}$$

$$= \frac{\beta^2 n}{(1+T\beta)^2}$$

$$= \frac{V(\lambda)}{(1+T\beta)}.$$

The Bayes risk approaches zero as the test time (T) approaches infinity.

DERIVATION OF THE UNCERTAINTY DISTRIBUTION AND ITS PARAMETER

The uncertainty distribution is derivable from Bayes theorem. For our purposes, this theorem is

$$g(\lambda|r, T) = f(r|\lambda)\, g(\lambda)/ f(r).$$

where $g(\lambda|r, T)$ is the uncertainty probability distribution of λ conditional on the sample results of r and T; $f(r|\lambda)$ is the probability of r failures in the life test, given the unit failure rate is λ ; $g(\lambda)$ is the <u>a priori</u> distribution of λ; and $f(r)$ is the unconditional distribution of r:

$$f(r) = \int_{\lambda} f(r|\lambda)\, g(\lambda)\, d\lambda.$$

By substitution of the known distributions on the right-hand side of the first expression above, the uncertainty probability distribution can be written as

$$g(\lambda|r, T) = \frac{\lambda^{(n+r)-1}\, e^{-\lambda\left(\frac{1+T\beta}{\beta}\right)}}{\left[(n+r)-1\right]!\ \frac{\beta}{1+\beta}\ (n+r)}, \quad \lambda > 0.$$

The uncertainty distribution is a gamma with expectation and variance respectively of

$$E(\lambda|r,T) = \left(\frac{\beta}{1+T\beta}\right) (n+r),$$

$$V(\lambda|r,T) = \left(\frac{\beta}{1+T\beta}\right)^2 (n+r).$$

The similarity between the first two moments of the uncertainty distribution and of the a priori distribution should be noted. The scaling parameter β of the a priori distribution corresponds to $\beta/(1+T\beta)$ in the uncertainty distribution. The shaping parameter n of the a priori distribution corresponds to (n+r) in the uncertainty distribution.

PRIORITY OF TESTING

As units of a similar type undergo testing, the reliability information known about these units changes and the value of the coefficient of variation (CV) for the system failure rate distribution, in turn, will be expected to decrease. In Chapter III it was established that if units of the i^{th} type were tested a total of x_i hours and units of the j^{th} type for a total of x_j hours, then these two tests would be expected to decrease the system CV an equal amount if x_i and x_j were related as,

$$x_i = \frac{\beta_j V(\lambda_j) \; x_j}{\beta_i V(\lambda_i) - \beta_i \beta_j \; [V(\lambda_j) - V(\lambda_i)] \; x_j} ,$$

or

$$x_j = \frac{\beta_i V(\lambda_i) \; x_i}{\beta_j V(\lambda_j) - \beta_i \beta_j \; [V(\lambda_i) - V(\lambda_j)] \; x_i} .$$

The test of the i^{th} units can be said to have a higher test priority if $o < x_i < x_j$, when solved for in the above expression. Similarly, if $o < x_j < x_i$, then the j^{th} units would be given the higher test priority.

While a test of one unit type may have a higher test priority initially, a point in total test time may be reached where the contribution of further testing has a less pronounced effect on decreasing the system CV. At this point it may pay to stop testing these units and begin testing of some other units.

Assume the i^{th} units have the higher test priority, then $o < x_i < x_j$ and from the second equality,

$$o < x_i < \frac{\beta_i V(\lambda_i)\ x_i}{\beta_j V(\lambda_j) - \beta_i \beta_j [V(\lambda_i) - V(\lambda_j)]\ x_i} .$$

After some manipulation we have,

$$o < x_i < \frac{\beta_i V(\lambda_i) - \beta_j V(\lambda_j)}{\beta_i \cdot \beta_j [V(\lambda_j) - V(\lambda_i)]} = H_{i,j} .$$

Therefore if the testing program calls for total testing of the i^{th} and j^{th} units such that

$$o < x_i,\ x_j < H_{i,j} ,$$

the test of the i^{th} units can be said to furnish more reliability information about the system.

In order that the j^{th} unit test have a higher priority, it is necessary that $o < x_j < x_i$ and by virtue of the first equality it follows that

$$o < H_{i,j} < x_j .$$

Therefore if the testing program calls for total testing of the i^{th} and j^{th} units such that

$$o < H_{i,j} < x_i , x_j$$

then test of the j^{th} units furnish more system reliability information.

To assign a test priority to units of the i^{th} and j^{th} type, it is necessary to compute the value of $H_{i,j}$. If the allocation of time to testing of the i^{th} and j^{th} units is less than $H_{i,j}$ then the i^{th} units would be given the higher test priority. If the allocation to both units is greater than $H_{i,j}$ then the j^{th} units would have a higher test priority for times in excess of $H_{i,j}$. If the test time assigned to both units are equal to $H_{i,j}$, then the tests of both units would be given equal priority. If $H_{i,j}$ is negative, the j^{th} unit will always have the greater test priority.

To assign a test priority between the i^{th} units and the system, a similar argument is used. The quantity $H_{i,s}$ is computed where,

$$H_{i,s} = \frac{\beta_i V(\lambda_i) - \beta_s V(\lambda_s)}{\beta_i \beta_s \quad [V(\lambda_s) - V(\lambda_i)]} .$$

If the test times x_i and x_s are less than $H_{i,s}$ then the i^{th} units are assigned the higher test priority while if the test times are greater than $H_{i,s}$, the system test is given the higher priority. Since the denominator of $H_{i,s}$ is always positive, it follows that if

$$\beta_s \; V(\lambda_s) > \beta_i \; V(\lambda_i) ,$$

the system will always have the higher test priority.

APPENDIX B

AN EXAMPLE: DETERMINING DEVELOPMENT RISKS AND THE ALLOCATION OF TESTING

APPENDIX B

AN EXAMPLE: DETERMINING DEVELOPMENT RISKS AND THE ALLOCATION OF TESTING

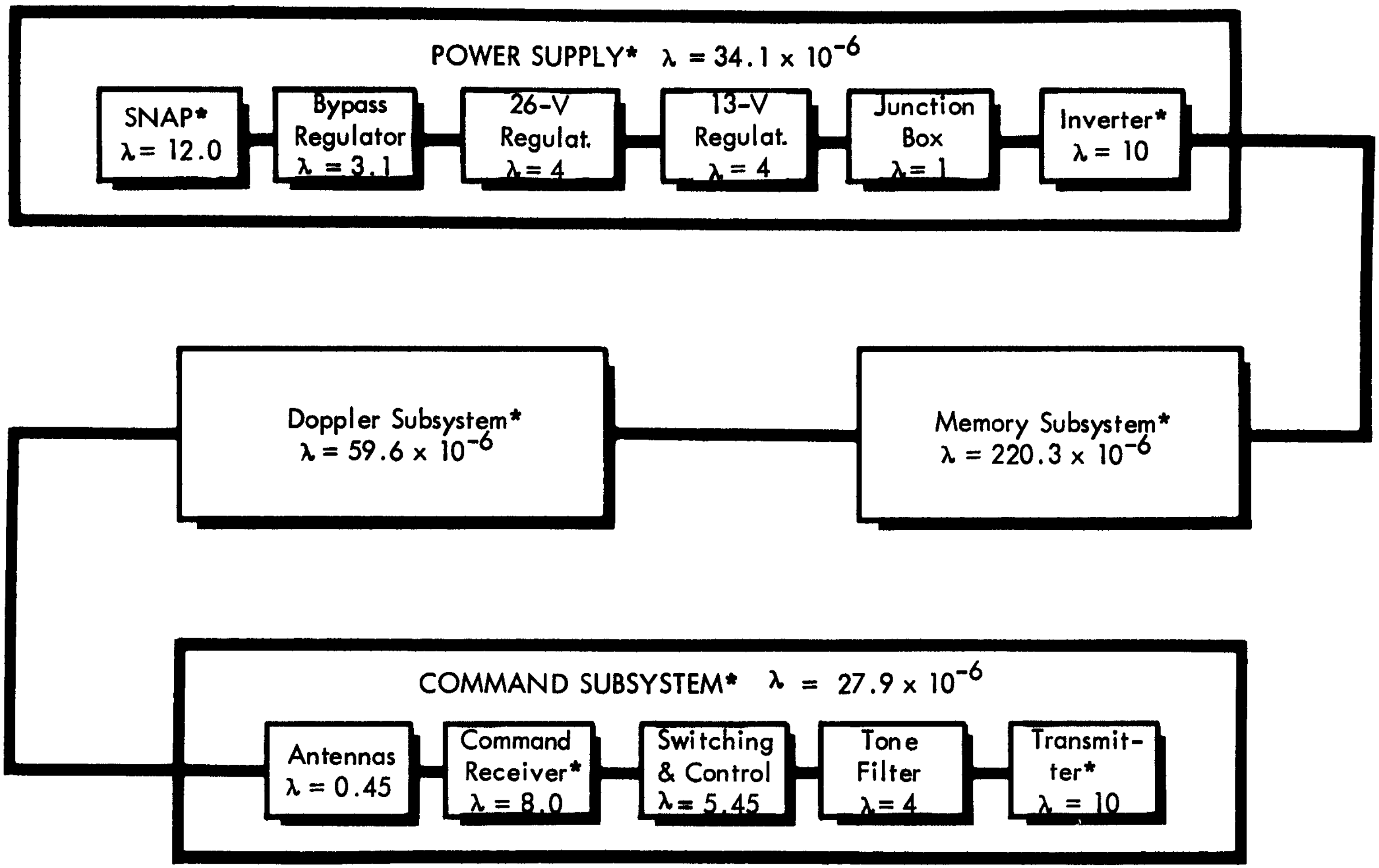
The following numerical examples illustrate the use of the formulas developed in the text on a fictitious spacecraft development. The system is represented by a Reliability Block Diagram showing the estimated failure rates. Information contained in the block diagram is similar to that prepared in the conventional reliability analysis.

The System Work Sheet presents all the basic input data needed for the system computations described in this report. The estimated unit variances would be derived by the technique shown in Appendix C. The information on the number of items which will be available for test, the test times, and the costs would be determined by the limitations imposed by the development program and its schedule.

Two basic analyses are made in this appendix. The first is the computation of the system reliability risk. This probability is computed first on the basis of system information known before testing, then on the basis of the expected gain in information as a result of testing, and finally on the basis of actual test results.

RELIABILITY BLOCK DIAGRAM

Units and subsystems marked by * can be tested individually and are available for testing. The unit failure rates are times 10^6.



System Worksheet #1

Testable Units/Subsystem*	System Information				Test Allocation Information		
	Estimate of Failure Rate (10^{-6})		Estimate of Variance (10^{-12})		Number of Test Items (Maximum)	Allowable Test Time/ Item - Hours	Test Cost/ Item
	Unit	Subsystem	Unit	Subsystem			
SNAP	12.0		128.0		1	4200	$ 50,000
Inverter	10.0		130.0		5	4200	$ 3,500
Power Supply		34.1		391.0	1	4200	$ 75,000
Memory		220.3		288.0	2	4200	$100,000
Doppler		59.6		262.0	2	3500	$ 15,000
Command Rec.	8.0		9.0		2	2800	$ 4,000
Transmitter	10.0		11.0		2	2800	$ 6,000
Command		27.9		33.0	2	2100	$ 18,000
System		341.9		974.0	1	2100	$300,000

* Only testable units are included in this table. The sum of unit failure rates and variances will not add to the subsystem failure rate and variance in all cases. The system failure rate and variance are the sum of the subsystem failure rates and variances.

The system reliability risk is dependent on the system reliability goal which, for this system, is stated as a 75 percent chance of survival of the spacecraft in orbit for one month.

The other analysis is concerned with the determination of the optimum, initial allocation of unit, subsystem, and system testing at the start of the program. A reallocation of test effort would take place at the completion of the first test. The initial allocation illustrated is based on an available $200,000 for reliability demonstration testing.

The numerical examples are typical of the types of analyses which would be made at the beginning of a development program (after the reliability of the design had been estimated) and at each milestone in the program. Analyses similar to those shown can be made at any point in the program when there is reason to change any of the input data--for example, following a design change, after testing, or due to changes in the program schedule or costs. The work sheets and computational methods would be the same.

In most reliability programs, it will be necessary to pursue the analysis to a greater depth than indicated by this appendix. Questions concerning the causes and correction of unreliability will require similar detailed analysis at the subsystem and component levels. These techniques can be used in planning of the development program to determine a reasonable amount of money to be spent on reliability demonstration testing in terms of the amount of information (change in reliability risks) which can be bought. If it becomes obvious that there can never be enough testing in the program to give sufficient control of the risks, then there is a justification to try to develop a more reliable system design.

DEFINITIONS USED IN APPENDIX B

λ_s	System failure rate.
λ_c	Critical failure rate corresponding to reliability goal.
λ_i	Failure rate of i^{th} unit.
$E(\lambda_s)$	Expected failure rate for system.
$E(\lambda_i)$	Expected failure rate for i^{th} unit.
$E(r_s)$	Expected number of system failures.
$E(\lambda \mid r, T)$	Expected failure rate following T hours of testing where failures are observed.
$V(\lambda_s)$	Variance of failure rate for system.
$V(\lambda_i)$	Variance of failure rate for i^{th} unit.
$V(\lambda \mid r, T)$	Variance of failure rate following T hours of testing where failures are observed.
β_s	Scale parameter for system failure rate distribution.
β_i	Scale parameter for i^{th} unit failure rate distribution.
β'_s	Estimates of β_s after unit testing.
$\beta_s(Y)$	Expected value of β_s after Y hours of system testing.
n_s	Shape parameter of system failure rate distribution.
n'_s	Estimates of n_s after unit testing.

$n_s(Y)$	Expected value of n_s after Y hours of system testing.
$\chi^2_{2n_s}$	Chi-square with $2n_s$ degrees of freedom.
u_{1-p}	Standard normal deviate.
$1-p$	Risk probability--the chance the system will fail to meet reliability goal.
P( )	Probability statement.
R( )	Reliability--probability of survival for a given period of time.
e	2.718 . . .
$\ln$	Logarithm to base e.
w_i	Weighting factor converting actual test hours to equivalent system hours.
C_o	Total money allocated for program testing.
Y	Total test time expressed in equivalent system test hours.
y_i	Actual test time for i^{th} unit.

INITIAL RELIABILITY RISK

Formula

The formula for computing the initial reliability risk is

$$P(\lambda_s > \lambda_c) = P\left(\chi^2_{2n_s} > \frac{2\lambda_c}{\beta_s}\right) = \text{risk.}$$

Computation of λ_s

The information is given in System Work Sheet #1.

Computation of λ_c

The reliability requirement is a 75 percent chance of system survival in orbit for one month.

$$R(\text{one month}) = e^{-\lambda_c (30 \text{ days} \times 24 \text{ hours})} = 0.75.$$

$$\lambda_c = \frac{-\ln 0.75}{30 \times 24}$$

$$= \frac{0.28768}{30 \times 24}$$

$$= 399.56 \times 10^{-6}$$

Computation of β_s and n_s

$$\beta_s = V(\lambda_s) \; / \; E(\lambda_s)$$

$$= 974.0 \times 10^{-12} \; / \; 341.9 \times 10^{-6}$$

$$= 2.849 \times 10^{-6}$$

$$n_s = E^2(\lambda_s) \; / \; V(\lambda_s)$$

$$= (341.9 \times 10^{-6})^2 \; / \; 974.0 \times 10^{-12}$$

$$= 120.$$

Computation of Risk Probability

$$\text{Risk} = P\left\{\chi^2_{2n_s} > \frac{2\lambda_c}{\beta_s}\right\}$$

$$= P\left\{\chi^2_{240} \quad \frac{799.12 \times 10^{-6}}{2.849 \times 10^{-6}}\right\}$$

$$= P\left\{\chi^2_{240} > 280.5\right\}.$$

Because values of chi-square (χ^2) are ordinarily not tabulated beyond 100 degrees of freedom (d.f=$2n_s$), the normal approximation to the chi-square distribution is used. Compute the standard normal deviate, u_{1-p}:

$$u_{1-p} = \sqrt{2\chi^2_{2n_s}} - \sqrt{2(2n_s)-1},$$

where (1-p) corresponds to the risk probability.

$$1-p = P(\lambda_s > \lambda_c) = \text{reliability risk.}$$

$$u_{1-p} = \sqrt{(2)(280.5)} - \sqrt{(2)(240)-1}$$

$$= 23.685 - 21.886$$

$$= 1.799$$

The actual reliability risk for the system, corresponding to $u_{1-p} = 1.799$, is 0.03593, as determined from tables of the cumulative normal distribution function.

INITIAL OPTIMUM ALLOCATION OF TEST EFFORT

Computation of Weighting Factors

To convert actual unit test hours to equivalent system test hours, compute the weighting factor.

$$w_i = \frac{\beta_i V(\lambda_i)}{\beta_s V(\lambda_s)} = \frac{V^2(\lambda_i) / E(\lambda_i)}{V^2(\lambda_s) / E(\lambda_s)}$$

For example, for the SNAP unit,

$$w_{SNAP} = \left[\frac{128 \times 10^{-12}}{974 \times 10^{-12}}\right]^2 \left[\frac{341.9 \times 10^{-6}}{12 \times 10^{-6}}\right] = 0.54 .$$

The weight means that a one-hour test of the SNAP is equivalent in information gain to 0.54 hours of system test.

Preparation of Table

Prepare a table of information gain and test cost for each testable unit, subsystem, and system.

Unit or Subsystem	No. of Items Tested	Total Hours of Test	w_i	Equiv. System Hours	Total Test Costs
SNAP	1	4200	0.54	2260	$ 50,000
Inverter	1	4200	0.61	2550	$ 3,500
	2	8400	0.61	5100	7,000
	3	12600	0.61	7650	10,500
	4	16800	0.61	10200	14,000
	5	21000	0.61	12750	17,500
Power Supply	1	4200	1.61	6750	$ 75,000
Memory	1	4200	0.137	574	$100,000
	2	8400	0.137	1148	200,000
Doppler	1	3500	0.42	1450	$ 15,000
	2	7000	0.42	2900	30,000
Command Rec.	1	2800	0.004	10	$ 4,000
	2	5600	0.004	20	8,000
Transmitter	1	2800	0.004	12	$ 6,000
	2	5600	0.004	24	12,000
Command	1	2100	0.013	28	$ 18,000
	2	4200	0.013	56	36,000
System	1	2100	1.0	2100	$300,000

<u>Optimum Allocation Procedure</u>

(a) From the preceding table, select the test with the highest equivalent system hours which is less in cost than the total dollars allowed for testing, C_o.

Total Test Money, C_o = \$200,000.

Select five inverters for test.

Equivalent System Hours = 12,750.

Cost = \$17,500.

(b) Compute the remaining total test money and from the remaining test units, not including inverters, repeat step (a).

Remaining Test Money = \$200,000 - \$17,500

= \$182,500.

Select one power supply.

Equivalent System Hours = 6,750

Cost = \$75,000.

(c) Repeat step (b) until all test money has been used.

(d) Optimum Allocation of $200, 000

Units Tested	#	Equivalent System Hours	Cost
Inverters	5	12, 750	$ 17, 500
Power Supply	1	6, 750	75, 000
Doppler	2	2, 900	30, 000
SNAP	1	2, 260	50, 000
Command	1	28	18, 000
Command Rec.	2	20	8, 000
		24, 708	$198, 500

EXPECTED RELIABILITY RISK AFTER TESTING

The information equation $\left(Y = y_s + \frac{\beta_i V(\lambda_i)}{\beta_s V(\lambda_s)} \; y_i + \ldots\right)$

is used to compute the total equivalent system test hours. As determined by the initial allocation, the equivalent hours are,

$$Y = 12,750 + 6750 + \ldots. + 20 = 24,708.$$

Compute the expected values of β_s and n_s.

$$\beta_s(Y) = \frac{\beta_s}{1 + Y\beta_s}$$

$$= \frac{2.849 \times 10^{-6}}{1 + (24,708)(2.849) \times 10^{-6}}$$

$$= \frac{2.849 \times 10^{-6}}{1.07039}$$

$$= 2.662 \times 10^{-6}$$

$$n_s(Y) = n_s + E(r_s)$$

The expected value of r_s, the number of system failures for system test time Y is,

$$E(r_s) = Y \; E(\lambda_s)$$

$$= (24,708)(341.9 \times 10^{-6})$$

$$= 8.45 .$$

$$n_s(Y) = 120 + 8.45$$

$$= 128.45 .$$

The expected reliability risk is

$$P\left\{\chi^2_{2n_s(Y)} > \frac{2\lambda_c}{\beta_s(Y)}\right\}$$

$$= P\left\{\chi^2_{257.90} > \frac{799.12 \times 10^{-6}}{2.662 \times 10^{-6}}\right\}$$

$$= P\left\{\chi^2_{257.90} > 300.2\right\} .$$

$$u_{1-p} = \sqrt{(2)(300.2)} - \sqrt{(2)(257.9)-1}$$

$$= 24.503 - 22.689$$

$$= 1.814$$

The expected risk after testing is 0.03484.

Computation of Actual Reliability Risk After Testing

The five inverters are tested for 4200 hours each (total of 21,000 hours). Three failures are observed. The estimate of inverter failure rate and the variance of the estimate after testing are:

$$E(\lambda_i \mid r, T) = \frac{E^2(\lambda_i) + r\,V(\lambda_i)}{E(\lambda_i) + T\,V(\lambda_i)} .$$

$$E(\lambda_i | 3; 21,000) = \frac{(10 \times 10^{-6})^2 + 3(130 \times 10^{-12})}{10 \times 10^{-6} + (21,000)(130 \times 10^{-12})}$$

$$= 38.49 \times 10^{-6} \ .$$

$$V(\lambda_i | r, T) = \frac{E^2(\lambda_i) + r\ V(\lambda_i)}{[E(\lambda_i) + T\ V(\lambda_i)]^2} \quad V(\lambda_i) \ .$$

$$V(\lambda_i | 3; 21,000) = \frac{(10 \times 10^{-6})^2 + 3(130 \times 10^{-12})}{\left[10 \times 10^{-6} + (21,000)(130 \times 10^{-12})\right]^2} \ 130 \times 10^{-12}$$

$$= 393.0810^{-12} \ .$$

The new estimate of system failure rate and the variance of that estimate become:

$$E(\lambda_s) | r, T) = E(\lambda_s) - E(\lambda_i) + E(\lambda_i / r, T) \ .$$

$$E(\lambda_s) | 3; 21,000) = (341.90 - 10.0 + 38.49)10^{-6} = 370.39 \times 10^{-6} \ .$$

$$V(\lambda_s | r, T) = V(\lambda_s) - V(\lambda_i) + V(\lambda_i / r, T) \ .$$

$$V(\lambda_s) 3; 21,000) = (974.0 - 130.0 + 393.08)10^{-12} = 1237.08 \times 10^{-12}.$$

Computation of system reliability risk after inverter test is:

$$\text{Risk} = P(\chi^2_{2n'_s} > 2\lambda_c/\beta'_s) \ .$$

Estimates of β_s and n_s after unit testing are:

$$\beta'_s = V(\lambda_s | r, T)/E(\lambda_s | r, T) \ .$$

$$\beta'_s = \frac{1237.08 \times 10^{-12}}{370.39 \times 10^{-6}} = 3.340 \times 10^{-6} \ .$$

$$n'_s = E^2(\lambda_s | r, T)/V(\lambda_s | r, T) \ .$$

$$n'_s = \frac{(370.39 \times 10^{-6})^2}{1237.08 \times 10^{-12}} = 110.9 \ .$$

$$\text{Risk} = P\left\{\chi^2_{2(110.9)} > \frac{799.12 \times 10^{-6}}{3.340 \times 10^{-6}}\right\}$$

$$= P\left\{\chi^2_{221.8} > 239.3\right\} .$$

By the normal approximation,

$$u_{1-p} = \sqrt{2(239.3)} - \sqrt{2(221.8)-1}$$

$$= 0.840 \ .$$

The actual risk after testing is 0.2005.

REALLOCATION

This reliability risk is excessive and changes in the development program should be considered. Additional inverter testing should be considered in order to reduce the risk.

Assume that five more inverters could be tested at $3,500 each. However, only 21,000 hours are available to conduct the test. A new work sheet would be prepared, making the appropriate change in the entries for inverters, power supply, and system.

New weighting factors are computed for each unit and subsystem in order to convert actual test hours into equivalent system test hours. A table of information gain and test cost is then prepared.

The allocation procedure is then repeated. Since $17,500 has already been spent on testing, C_o = $182,500 instead of $200,000. The new optimum allocation is as follows:

Units Tested	#	Equivalent System Hours	Cost
Inverters	5	10,185	$ 17,500
Power Supply	1	6,930	75,000
Doppler	2	1,960	30,000
SNAP	1	1,386	50,000
Command Rec.	2	14	8,000
		20,475	$180,500

The expected risk after testing is 0.1936.

System Worksheet #2

Testable Units/Subsystem*	System Information				Test Allocation Information		
	Estimate of Failure Rate (10^{-6})		Estimate of Variance (10^{-12})		Number of Test Items (Maximum)	Allowable Test Time/ Item - Hours	Test Cost/ Item
	Unit	Subsystem	Unit	Subsystem			
SNAP	12.0		128.0		1	4200	$ 50,000
Inverter	38.49		393.08		5	2100	$ 3,500
Power Supply		62.59		654.08	1	4200	$ 75,000
Memory		220.3		288.0	2	4200	$100,000
Doppler		59.6		262.0	2	3500	$ 15,000
Command Rec.	8.0		9.0		2	2800	$ 4,000
Transmitter	10.0		11.0		2	2800	$ 6,000
Command		27.9		33.0	2	2100	$ 18,000
System		370.39		1237.08	1	2100	$300,000

* Only testable units are included in this table. The sum of unit failure rates and variances will not add to the subsystem failure rate and variance in all cases. The system failure rate and variance are the sum of the subsystem failure rates and variances.

Table of Information Gain and Test Cost

Unit or Subsystem	No. of Items Tested	Total Hours of Test	w_i	Equiv. System Hours	Total Test Costs
SNAP	1	4200	.33	1386	$ 50,000
Inverter	1	2100	.97	2037	$ 3,500
	2	4200	.97	4074	7,000
	3	6300	.97	6111	10,500
	4	8400	.97	8148	14,000
	5	10500	.97	10185	17,500
Power Supply	1	4200	1.65	6930	$ 75,000
Memory	1	4200	.09	382	$100,000
	2	8400	.09	764	200,000
Doppler	1	3500	.28	980	$ 15,000
	2	7000	.28	1960	30,000
Command Rec.	1	2800	.0025	7	$ 4,000
	2	5600	.0025	14	8,000
Transmitter	1	2800	.0029	8.1	$ 6,000
	2	5600	.0029	16.2	12,000
Command	1	2100	.0094	19.7	$ 18,000
	2	4200	.0094	39.5	36,000
System	1	2100	1.0	2100	$300,000

APPENDIX C

FAILURE RATE VARIABILITY MODEL

APPENDIX C

FAILURE RATE VARIABILITY MODEL

The approach described in this report is based on three concepts:

(1) Reliability can be predicted, using historical experience and engineering analysis.

(2) Reliability can be measured, using test data.

(3) Each prediction and each measurement has some degree of uncertainty because of inherent variability and sampling errors.

In general, these uncertainties account for the inadequacy of predictions and tests for the purposes of reliability demonstration. Our approach seeks to combine the estimates obtained from prediction and test in order to give a degree of precision and assurance which neither source can furnish alone.

This appendix describes a subsidiary model which, by analyzing the variability of historically-based failure rate data as well as their value, extends the conventional techniques of assessment and prediction to the estimation of uncertainty. Uncertainty estimates are required if prediction and test results are to be combined.

Methods for assessing the effects of deviation from "normal practice" are presented, and examples of procedure and computation are provided. Additional applications of the model are also discussed.

FAILURE RATE VARIABILITY

Sources of Failure Rate Variability

In reliability prediction, it is accepted practice to obtain individual part failure rates from standard tables, modify these failure rates to reflect stress levels, and assign these adjusted rates as the applicable failure rates for parts.

The failure rates quoted in standard tables are large-sample averages obtained from field and laboratory experience with equipments and systems. For each part type, the quoted failure rate is an average based on many manufacturing lots from a variety of parts manufacturers and reflecting usage representing the entire spectrum of design sophistication, workmanship, and maintenance practices. Known differences in application stresses are accounted for by trade-off curves of a deterministic nature, so that experience data may be converted to standard conditions and pooled.

A standard failure rate obtained from standard tables may then be regarded as a close approximation to the true mean failure rate for that part type as used in equipment under standard conditions. The failure rate obtained for an individual part by applying deterministic application stress factors to the mean failure rate will be termed a standard failure rate and will be denoted by λ_{ijk}, with additional subscripts as needed.

Part failure rates obtained by observation of equipment and system operation contain major contributions from design, packaging, and assembly inadequacies, as well as from inherent deficiencies in the parts themselves. This is true even when the raw data are edited to remove such gross errors as incorrect wiring. Part failure due to improper handling, circuit susceptibility to predictable drift, or inadequate isolation from mechanical or thermal shock will generally be recorded as a failure of the part involved and reflected in the failure rate tabulations.

Pooled data from a variety of sources indicate that, on the average, 35 percent of the observed part failures are due to inherent parts deficiencies, 35 percent to design inadequacies (including susceptibility to predictable part parameter drift), 10 percent to packaging problems (such as inadequate protection or isolation from shock), and 20 percent to assembly effects (such as improper soldering undetected in inspection or final checkout).

Thus, the standard failure rate (λ_{ijk}) for any part is the sum of four contributors--$.35\lambda_{ijk}$ due to parts, $.35\lambda_{ijk}$ due to design, $.1\lambda_{ijk}$ due to packaging, and $.2\lambda_{ijk}$ due to assembly--each of which is a standard value for its category.

While λ_{ijk} is a fixed known value, each of the contributors is a random variable. When a contractor's practice is normal, the standard failure rate is the expected value. There is also an uncertainty, which can be expressed in terms of a variance, dependent on such influences as lot-to-lot and vendor-to-vendor variability in parts, accidental differences in quality of design, and variations in assembler and inspector skill and training among contractors.

When standard failure rates are combined to yield a failure rate estimate for an equipment or system, the result obtained is again a mean. It is a mean, however, for a hypothetical population of such equipments or systems in which the entire range of parts manufacturers and lots, design sophistication, workmanship, and maintenance practices is represented. Such a population can only be approximated with equipments or systems in large production. Typical space systems, subsystems, and equipments should not be expected to provide observed failure rates which are close to the predicted (mean) failure rate, because deviations of the kind noted above are likely to be significant.

Because it is highly probable that a single lot from a single vendor will provide all parts of a given type when equipment production is small, averaging effects cannot be expected and chance deviations due to a single "bad" lot may have a large influence on the observed failure rate. This has been considered in a more limited study of the variability of failure rates.*

Thus far, only chance deviations which may be anticipated under conditions of normal practice have been considered. It is possible that systematic differences will occur when the contractor's practice systematically differs from the normal. Thus, if a contractor employs parts screening specifications and a

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* "The Uncertainty of Reliability Assessments," G. R. Herd, R. L. Madison, P. Gottfried; Booz, Allen Applied Research Inc., Tenth National Symposium on Reliability and Quality Control, January 1964.

vendor approval program, designs circuits with the aid of a computer simulation technique, conducts formal reviews of packaging design, and applies well-controlled automated assembly techniques, his product will display failure rates lower than the normal-practice mean.

If another contractor uses commercial parts not qualified to government specifications, employs inexperienced design engineers, and relies on untrained part-time employees on his assembly line, higher-than-normal failure rates will be observed. Expected values of individual part failure rates should be adjusted accordingly, and estimates of variance should be modified to reflect the reduced range of probable variation which accompanies more detailed knowledge of the individual contractor's practices as compared to the norms for the industry.

It is necessary to structure any model for accumulation of variances so as to account for independence and interdependence among and within groupings. If, as previously suggested, all parts of one type originate in one lot, individual parts of one type will not be independent with respect to inherent deficiencies. Such independence will be retained among the several part types used. Grouped in another way, all parts--regardless of type--used within a single circuit will be subjected to the same design influence and will be interdependent in this respect. Different circuits in the same equipment may vary in actual quality of design and hence are independent.

Similarly, packaging effects must be considered uniform within a black box, but not among black boxes. A larger assembly, which we shall call a unit (equipment or subsystem), can be expected to subject the parts contained therein to the same assembly line and inspection influences. In this respect, the failure rate contributions within such a unit will be interdependent. Thus, it will be necessary to group and sum in four ways, corresponding to the four major contributors, in order to compute variance in failure rates at the unit level. For consistency, the model for computation of expected value of unit failure rate will be structured correspondingly.

A Model for Combining Failure Rates and Variances

Consider a unit (equipment, subsystem) containing N individual parts. These parts may be grouped in N_i part types in N_j circuits, or in N_k black boxes. We define $T_{i..}$ to be the sum of the standard failure rates of the parts contained in the i^{th} part type, $T_{.j.}$ the sum of the standard failure rates of the parts contained in the j^{th} circuit, and $T_{..k}$ the sum of the standard failure rates of the parts contained in the k^{th} black box.

The individual parts in the unit are in a series relationship, and the summation over all parts or over any grouping of parts is equivalent to the unit. The predicted value of the failure rate of the unit under conditions of normal practice (λ_u) can be expressed as:

$$\lambda_u = \sum_{i=1}^{N_i} T_{i..} \equiv \sum_{j=1}^{N_j} T_{.j.} \equiv \sum_{k=1}^{N_k} T_{..k} \equiv T_{...} \quad .$$

As defined earlier, λ_{ijk} is composed of four contributors--$.35\,\lambda_{ijk}$ due to inherent parts deficiencies, $.35\,\lambda_{ijk}$ due to design inadequacies, $.1\,\lambda_{ijk}$ due to packaging problems, and $.2\,\lambda_{ijk}$ due to assembly effects. The parts deficiency contributions may be summed within and over part types, the design contributions within and over circuits, the packaging contribution within and over black boxes, and the assembly contributions over individual parts, obtaining

$$\lambda_u = \sum_{i=1}^{N_i} .35T_{i..} + \sum_{j=1}^{N_j} .35T_{.j.} + \sum_{k=1}^{N_k} .1T_{..k} + .2T_{...}.$$

This arbitrarily chosen form contains groupings which conform to the interdependence/independence requirements noted previously. For example, the packaging contributions of all parts which are subject to common packaging effects (by virture of being contained in the same black box) are grouped in $T_{..k}$.

To obtain a more general form, a parts factor is defined as (a), a design factor (b), a packaging factor (c), and an assembly factor (d). The expected value of each factor corresponds to the relative contribution noted above, so that $E(a) = .35$, $E(b) = .35$, $E(c) = .1$, and $E(d) = .2$. We then define weighting factors W_a, W_b, W_c, and W_d, and ratios p_a, p_b, p_c, and p_d, such that

$$W_a = p_a E(a) = .35\, p_a,$$

$$W_b = p_b E(b) = .35\, p_b,$$

$$W_c = p_c E(c) = .1\, p_c,$$

$$W_d = p_d E(d) = .2\, p_d,$$

where each weighting factor and each ratio is a random variate.

Then

$$E(W_a) = E(p_a)\,E(a)$$

and
$$V(W_a) = V(p_a)\,E^2(a).$$

and similarly for each of the other weighting factors and ratios.

Where normal practice prevails, a single value of $E(p_a)$ and a single value of $V(p_a)$ apply to all part types, and similarly unique expected values and variances apply to each other set of groupings. Since we wish the expected value for each failure rate and failure rate contribution to equal the corresponding standard value when normal practice prevails,

$$E(p_a) = E(p_b) = E(p_c) = E(p_d) = 1$$

under those conditions.

With departure from normal practice, we wish to allow for differences in $E(p_a)$ and $V(p_a)$ among (but not within) part types, in $E(p_b)$ and $V(p_b)$ among (but not within) circuits, and in $E(p_c)$ and $V(p_c)$ among (but not within) black boxes. We may denote these values with subscripts corresponding to the groupings involved--$E(p_{a_i})$, $V(p_{c_k})$. Single values of $E(p_d)$ and $V(p_d)$ continue to apply within the entire unit, but it is no longer necessary that $E(p_d) = 1$. In general, the expected value of the unit failure rate (λ_u) may be expressed as:

$$E(\lambda_u) = \sum_{i=1}^{N_i} E(p_{a_i}) \; .35T_{i..} + \sum_{j=1}^{N_j} E(p_{b_j}) \; .35T_{.j.}$$

$$+ \sum_{k=1}^{N_k} E(p_{c_k}) \; .1T_{..k} + E(p_d) \; .2T_{...}$$

and its variance as:

$$V(\lambda_u) = \sum_{i=1}^{N_i} V(p_{a_i}) \, (.35T_{i..})^2 + \sum_{j=1}^{N_j} V(p_{b_j}) \, (.35T_{.j.})^2$$

$$+ \sum_{k=1}^{N_k} V(p_{c_k}) \, (.1T_{..k})^2 + V(p_d) \, (.2T_{...})^2 \quad .$$

A basis for estimating the expected value and variances of the ratios required for numerical solution of these equations is presented next.

FAILURE RATE ADJUSTMENT RATIOS

Variance under Conditions of Normal Practice

Each of the four ratios, p_a, p_b, p_c, and p_d, is a composite value arising from a number of individual contributory causes. It is known, for example, that lot-to-lot variability and vendor-to-vendor variability are responsible for wide variations in observed failure rates, and that control measures such as screening specifications and formal vendor approval can reduce the average failure rate observed and also the range of variation. We need to estimate the variance arising from the relative lack of control prevailing in normal practice, and the modifications of expected value and variance to be expected when the practice deviates--favorably or unfavorably--from normal. The ratios then express the relationship of failure rates under deviant practices to those under normal practices.

The individual contributors to each of the four ratios may combine in additive, multiplicative, or other manner. A multiplicative relationship, such that each of the ratios p_a, p_b, p_c, and p_d is the product of some set of contributors, is convenient for purposes of evaluation and computation. This type of relationship is logical because one expects the total failure incidence level to vary with process control and inspection technique levels as multiplying factors, rather than as additive or subtractive increments. It is consistent with the uniform observation of skewness in the distribution of failure rate with respect to those parameters--lot-to-lot variability, vendor-to-vendor variability, variability among field sites--which have been investigated in some detail (see reference 1).

In fact, the examples cited show good fits when log-normal distributions are applied. The fact that the log-normal distribution has reproductive properties in multiplication, and that this distribution arises theoretically when the effect of a contributing factor is related to the magnitude of the quantity acted upon, constitutes further support for our choice of model.

On the basis of laboratory and field data, we expect to observe a 100:1 variation in the value of p_a among 40 groups of parts of one type. As noted above, we have chosen to attribute log-normal form to the distribution of each ratio.

Using the notation of Aitchison and Brown* with minor modifications, we define an essentially positive variate $X (0 < X < \infty)$, corresponding to each of the ratios, p_a, p_b, p_c, and p_d, such that $Y = \ln X$ is normally distributed with mean μ and variance σ^2.

We then write:

$$X \text{ is } L(\mu, \sigma^2)$$

and

$$Y \text{ is } N(\mu, \sigma^2) \quad .$$

$$E(x) = e^{\mu + \frac{1}{2}\sigma^2}$$

$$V(x) = e^{2\mu + \sigma^2} (e^{\sigma 2} - 1)$$

$$\eta^2 = e^{\sigma 2} - 1 \quad ,$$

where η is the coefficient of variation.

Since the 100:1 variation in a sample size of 40 corresponds to $40\sigma = \ln 100$, and $E(p_a) = 1$ for normal practice, we have a tolerance interval such that approximately 95 percent of all values of p_a are contained in the interval $.05 < p_a < 5.0$. We also obtain

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* "The Log-Normal Distribution," J. Aitchison and J. A. C. Brown, Cambridge University Press, 1957.

$\eta_a^2 = 2.74$. Therefore, $V(p_a) = 2.74$ for the normal practice case. In the absence of more definitive data, we assign the same values to the variance and coefficient of variation for each of the other ratios for normal practice.

Deviations from Normal Practice

Evaluation sheets similar to the samples contained in this appendix may be employed to estimate the results of deviations from normal practice. For each contributing cause, the level of practice is indicated, each level is weighted, and the level corresponding to normal practice is given the weight of 1. The product of the selected weights then provides an estimate of the expected value of the ratio associated with the evaluation sheet. An expected value may thus be assigned to each ratio for each specific situation. Of course, these estimates are tentative and are subject to modification as practical experience with the model is gained.

When evaluation of a unit (and the contractor responsible for its design and assembly) leads to modified estimates of the expected values $E(p_a)$, $E(p_b)$, $E(p_c)$, and $E(p_d)$, we should also anticipate a change in the corresponding variances. The modified estimates represent an increase in information, in the sense that it becomes known that the actual practice lies in a more restricted range than the range corresponding to normal practice. This is true whether the expected value is larger or smaller than the normal practice expected value. Accordingly, we wish to associate a smaller variance with a ratio when its expected value is not equal to 1. On logical grounds, we also require that the mode ($e^{\mu-\sigma^2}$) and the median (e^{μ}) move in the same direction as the expected value.

A convenient definition which satisfies these criteria is:

$$\frac{V(X_1)}{V(X)} = \left[\frac{E(X_1)}{E(X)}\right]^2 , \quad E(X_1) \leq E(X)$$

and

$$\frac{V(X_1)}{V(X)} = \left[\frac{E(X)}{E(X_1)}\right]^2 , \quad E(X_1) \geq E(X) \quad ,$$

where X is the variate for normal practice and X_1 is the variate for any other level. This states that the ratio of new to original variance is equal to the square of the ratio of the lesser expected value to the larger. For computational convenience, the values of variance corresponding to expected values obtainable from the evaluation sheets may be tabulated.

Preliminary Sketches of Evaluation Sheets

The evaluation sheets in this section are intended only to illustrate the factors to be considered. Further work will be required to assign numerical values to each factor and to refine the format.

Multiple choices for rating each factor will be provided, using hedonic scales when applicable, as illustrated in the fourth evaluation sheet. Each sheet will also include one or more "trap" questions as a check on the individual conducting the survey.

This individual should not be aware of the numerical value of his choices. Numerical evaluation of the survey can be conducted by clerical personnel, using key charts, after the evaluation sheets have been submitted.

It may be desirable to evaluate any bias in individual survey personnel. This may be accomplished by requiring all personnel to conduct a survey for a specific equipment item in a facility utilizing known practices. Results would then be compared. Normalization of survey personnel (correction of raw sources for known bias) may be considered.

PARTS

	Yes	No
Employs high reliability specs		
1. For hi-usage parts only	☐	☐
2. For virtually all parts (Except specials)	☐	☐
Employs substantial effort in parts screening tests	☐	☐
Employs contractor-prepared specs in effort to improve on MIL quality	☐	☐
Has and uses preferred parts list	☐	☐
Has parts application engineering staff	☐	☐
Routinely conducts parts application review (Formal, but not necessarily by specialists)	☐	☐
Maintains failure analysis laboratory	☐	☐
Maintains and enforces approved vendor list	☐	☐

DESIGN

	Yes	No
Uses selection of parts (e.g., select transistor for $35<B<50$ where type range is $25<B<70$)	☐	☐
Uses Matched Parts		
Matched for parameter	☐	☐
Matched for parameter and T&C	☐	☐
Matched for parameter, T&C, aging	☐	☐
Uses aging compensation	☐	☐
Uses feedback stab. techniques	☐	☐
Uses worst-case design	☐	☐
Uses Monte Carlo based design	☐	☐
Function isolation (Typical no. of interacting stages) Check one.	☐	☐

- 2-3 ☐
- 4-5 ☐
- 6-7 ☐
- 8-9 ☐
- 10 or more ☐

	Yes	No
Uses standard (preferred) circuits	☐	☐

Design Review effort. Check one.

- Intensive - well staffed ☐
- Good ☐
- Average ☐
- Weak ☐
- None ☐

PACKAGING

	Yes	No
Thermal		
Thermal Mass and/or insulation adequate to prevent thermal shock. Check one.		
Good ☐		
Normal ☐		
Poor ☐		
Parts placement. Check one.		
Good ☐		
Normal ☐		
Poor ☐		
Mechanical		
Isolators or structural damping	☐	☐
Conformal coating	☐	☐
Parts placement. Check one.		
Very good ☐		
Good ☐		
Poor ☐		
Very Poor ☐		
Atmospheric. Check one.		
Close-fitting, unsealed dust cover ☐		
Hermetic Seal ☐		
Potting ☐		

ASSEMBLY

	Very Good	Good	Normal	Poor	Very Poor
Process Control	☐	☐	☐	☐	☐
In-Process Inspection	☐	☐	☐	☐	☐
Final Inspection	☐	☐	☐	☐	☐
Training	☐	☐	☐	☐	☐

Method of Interconnection (Check only one method, that used primarily)

	Hand Soldering	☐
	Welding	☐
	Dip- or Flow-Soldering	☐
	Crimp	☐

☐ Check if most interconnections are redundant.

Principal Method of Assembly (Check only one)

	Hand Wired	☐
	Printed Circuit	☐
	"Cord Wood"	☐
	Micromodule	☐
	Microminiature (thin-film or integrated)	☐

EXAMPLE OF PROCEDURE

The application of the failure rate variability model will be practical only if routine procedures are established. For comparatively simple units or systems, manual processing may be applicable. By providing appropriate worksheets and step-by-step instructions, the computational tasks can be simplified so that they may be performed by clerical or junior-level personnel. For more complex units, computer processing may be desirable. General purpose programs can be prepared on the basis of step-by-step descriptions.

A specimen procedure is presented for application of the model to the estimation of failure rate expectation and variance. It is applicable at any stage after initiation of a contract to design and construct a unit. The depth of detail and the anticipated accuracy of the estimates will vary, depending on the state of advancement of the design and on the extent of the evaluation survey. These factors affect the amount of effort required to produce the estimates, but the procedure remains essentially the same.

Contractor Evaluation Procedure

1. Generate parts list.
   (May be arranged in any convenient manner--by circuit, part type, etc.)

2. Determine standard failure rate for each part, taking stresses into account.

3. Code each part to identify its location within a subdivision in each of three categories. (e.g., part type 3, circuit 6, black box 2)

4. Sum all failure rates within each subdivision, within each part type, within each circuit, and within each black box.

5. Sum failure rates over all parts, over all part types, over all circuits, and over all black boxes. The four sums should be equal to each other.

6. Obtain estimates of the four ratios (parts, design, packaging, assembly) from evaluation sheets. If this is a general evaluation, only one estimate for each ratio may be found. If it is a detailed evaluation, there may be estimates for each subdivision.

7. (a) Multiply the failure rate sum for each part type by 0. 35.

   (b) Multiply the failure rate sum for each circuit by 0. 35.

   (c) Multiply the failure rate sum for each black box by 0. 1.

   (d) Multiply the failure rate sum for all parts by 0. 2.

8. (a) Multiply the values obtained in 7. (a) by the corresponding estimates of the parts ratio, p_a.

   (b) Multiply the values obtained in 7. (b) by the corresponding estimates of the design ratio, p_b.

   (c) Multiply the values obtained in 7. (c) by the corresponding estimates of the packaging ratio, p_c.

   (d) Multiply the values obtained in 7. (d) by the estimate of the assembly ratio, p_d.

9. Sum the results obtained in 8. (a), 8. (b), and 8. (c). Add these three sums to the result obtained in 8. (d). This total is the estimate of the expected value of the unit failure rate.

10. Examine all circuits and black boxes and identify any which are identical in design, parts count, and configuration. Combine the values obtained in 7. (b) for identical circuits and those obtained in 7. (c) for identical black boxes.

11. (a) Square each value obtained in 7. (a).

    (b) Square each value obtained in 7. (b), except for circuits having identical counterparts. For each set of identical circuits, square the combined values in the set.

    (c) Square each value obtained in 7. (c), except for black boxes having identical counterparts. For each set of identical black boxes, square the combined values in the set.

    (d) Square the value obtained in 7. (d).

12. Look up or compute an estimate of $V(p_a)$ for each estimate of $E(p_a)$ $V(p_b)$ for $E(p_b)$, $V(p_c)$ for $E(p_c)$, and $V(p_d)$ for $E(p_d)$.

13. (a) Multiply each result obtained in 11. (a) by the corresponding estimate of $V(p_a)$.

    (b) Multiply each result obtained in 11. (b) by the corresponding estimate of $V(p_b)$,

    (c) Multiply each result obtained in 11. (c) by the corresponding estimate of $V(p_c)$.

    (d) Multiply the result obtained in 11. (d) by the estimate of $V(p_d)$.

14. Sum the results obtained in 13. (a), 13. (b), and 13. (c). Add these three sums to the result obtained in 13. (d). This total is the estimate of the variance of the unit failure rate.

COMPUTATION EXAMPLE

To obtain a realistic example, failure rate data for a small subsystem--the "Infrared Electronics" of the TIROS III satellite--has been extracted from an earlier BAARINC report.* Parts and standard failure rates by general part type within each circuit are listed in the report worksheets.

For the purposes of the failure rate variability model, part types have been defined as subclasses within general part types; e.g., 1/2-watt carbon composition resistors of nominal 100-ohm value are members of one part type, and similar resistors of 4,700-ohm nominal value constitute another part type. This definition is required for consistency with the assumption that all parts of one type originate in a single lot from one vendor.

This required level of detail was not retained in the TIROS worksheets. Therefore, each general part type has been subdivided to correspond to a reasonable number of values, and the recorded failure rate total for each general part type has been apportioned among these

* "Reliability Assessment of the TIROS Satellite," Task IV, Contract No. NASw-230, Booz, Allen Applied Research Inc., June 30, 1962.

part type subdivisions. The results are shown in Column $T_{i..}$ of Table C-1 for the entire subsystem. Similarly, the totals for each type of circuit are shown in Table C-2, and for each black box in Table C-3.

The next column in each table shows the computed normal practice contribution corresponding to the factor under consideration--parts effects in Table C-1, design effects in Table C-2, and packaging effects in Table C-3. The assembly effect contribution is obtained directly from the total failure rate for the subsystem (597.7×10^{-6}) multiplied by 0.2; i.e., $0.2T_{...} = 0.2\ E(\lambda_u)$.

The final column, which shows the computed square of the contribution, is required as an interim step in obtaining the variance. Most insignificant values have been omitted and many more could be excluded without significant effect on overall accuracy. Considerable computation effort can be avoided in this manner.

The variance due to each effect is now obtained, for the normal practice case, by multiplying the sum of squares (from the tables and for the assembly effect) by 2.74 (the value of η^2, the square of the coefficient of variation). These results, the total variance for the subsystem, the standard deviation for the subsystem, and the overall coefficient of variation for the subsystem are shown in Table C-4.

Table C-1
Standard Failure Rate Totals by Part Type

	$T_{i..}$ $\times 10^{6}$	$.35T_{i..}$ $\times 10^{6}$	$(.35T_{i..})^2$ $\times 10^{12}$
Capacitor, Ceramic, Type A	.025	.009	--
Type B	.020	.007	--
Mica, Type A	.120	.042	--
Type B	.100	.035	--
Type C	.050	.018	--
Paper, Type A	4.500	1.575	2.481
Type B	3.000	1.050	1.103
Type C	1.021	.357	.127
Tantalum, Wet, Type A	150.000	52.500	2756.250
Type B	125.000	43.750	1914.063
Type C	100.000	35.000	1225.000
Type D	30.020	10.507	110.397
Tantalum, Solid	14.000	4.900	24.010
Variable	.002	.001	--
Other	.450	.158	.025
Coil, Choke, Type A	.250	.088	--
Type B	.217	.076	--
Crystal, Quartz	.007	.002	--
Diode, Germanium, Type A	15.000	5.250	27.563
Type B	4.400	1.540	2.372
Type C	10.000	3.500	12.250
Diode, Silicon, Type A	3.250	1.138	1.295
Type B	.756	.265	.070
Diode, Zener, Type A	6.000	2.100	4.410
Type B	3.816	1.336	1.785
Filter (RLC)	.027	.009	--
Relay, Type A	10.000	3.500	12.250
Type B	5.000	1.750	3.063
Resistor, Carbon Composition, Type A	9.000	3.150	9.923
Type B	5.000	1.750	3.063
Type C	3.000	1.050	1.103
Type D	3.000	1.050	1.103
Type E	2.995	1.048	1.098
Deposited Film, Type A	.500	.175	.031
Type B	.500	.175	.031
Potentiometer, Type A	4.045	1.416	2.005
Type B	3.000	1.050	1.103
Type C	1.800	.630	.397
Other	1.606	.562	.316
Thermistor	.900	.315	.099
Transistor, Germanium, Type A	.600	.210	.044
Type B	.200	.070	--
Silicon, Type A	20.000	7.000	49.000
Type B	12.000	4.200	17.640
Type C	4.272	1.495	2.235
Type D	18.000	6.300	39.690
Type E	6.812	2.384	5.683
Transformer, Type A	1.000	.250	.123
Type B	.699	.245	.060
Tube, Electron, Type A	10.000	3.500	12.250
Type B	.075	.026	--
Type C	.375	.131	.017
Tuning Fork	.300	.105	.011
Totals	$\sum_{i=1}^{N_i} T_{i..} = 596.7 \times 10^{-6}$	$.35\sum_{i=1}^{N_i} T_{i..} = 208.8 \times 10^{-6}$	$\left(.35\sum_{i=1}^{N_i} T_{i..}\right)^2 = 6,246 \times 10^{-12}$

Table C-2

Standard Failure Rate Totals by Circuit

Circuit	Quantity	Failure Rate per cct. x 10^6	$T_{.j.}$ x 10^6	$.35T_{.j.}$ x 10^6	$(.35T_{.j.})^2$ x 10^{12}
Infra-Red Control Board	1	33.40	33.40	11.690	136.66
Subcarrier Oscillator	5	4.79	23.95	8.383	70.27
Playback Amplifier	1	0.44	0.44	.154	.02
Power Supply	1	150.97	150.97	52.840	2792.07
A.C. Sync. Motor Supply	1	22.24	22.24	7.784	60.59
D. C. Supply	1	0.55	0.55	.193	.04
Channel 6 Subcarrier Osc.	1	16.93	16.93	5.926	35.12
Channel 7 Tuning Fork Osc.	1	6.47	6.47	2.265	5.13
Transmitter	1	0.78	0.78	.273	.07
Type "H" Amplifier	2	55.41	110.82	38.787	1504.43
Type "T" Amplifier	5	46.03	230.15	80.553	6488.79

Totals $\sum_{j=1}^{N_j} T_{.j.} = 596.7 \times 10^{-6}$ $\quad .35\sum_{j=1}^{N_j} T_{.j.} = 208.8 \times 10^{-6}$ $\quad \left(.35\sum_{j=1}^{N_j} T_{.j.}\right)^2 = 11,093 \times 10^{-12}$

Table C-3
Standard Failure Rate Totals by Black Box

Black Box	$T_{..k}$ x 10^6	$.1T_{..k}$ x 10^6	$(.1T_{..k})^2$ x 10^{12}
Control Board	33.40	3.34	11.16
Data Channels (all)	388.32	38.83	1507.77
Playback Amplifier	0.44	.04	--
Power Supply	150.97	15.10	228.01
A.C. Sync. Motor Supply	22.24	2.22	4.93
D.C. Supply	0.55	.06	--
Transmitter	0.78	.08	--

$$\sum_{k=1}^{N_k} T_{..k} = 596.7 \times 10^{-6} \qquad .1\sum_{k=1}^{N_k} T_{..k} = 59.7 \times 10^{-6} \qquad \left(.1\sum_{k=1}^{N_k} T_{..k}\right)^2 = 1752 \times 10^{-12}$$

Table C-4
Variance

Variance due to parts effects	$V(a)\left(\sum_{i=1}^{N_i} .35T_{i..}\right)^2$	$= 2.74 \times 6246 \times 10^{-12}$	$= 17114 \times 10^{-12}$
Design effects	$V(b)\left(\sum_{j=1}^{N_j} .35T_{.j.}\right)^2$	$= 2.74 \times 11093 \times 10^{-12}$	$= 30395 \times 10^{-12}$
Packaging effects	$V(c)\left(\sum_{k=1}^{N_k} .1T_{..k}\right)^2$	$= 2.74 \times 1752 \times 10^{-12}$	$= 4800 \times 10^{-12}$
Assembly effects	$V(d)\ (.2T_{...})^2$	$= 2.74 \times 14256 \times 10^{-12}$	$= 39061 \times 10^{-12}$
		Total variance $V(\lambda_u)$	$= 91370 \times 10^{-12}$
		$\sigma_u = \sqrt{V(\lambda_u)}$	$= 302.3 \times 10^{-6}$
		$\eta_u = \dfrac{\sigma_u}{E(\lambda_u)} = \dfrac{302.3}{596.7}$	$= .507$

Sample Work Sheet

Drawing or Item Identification: Infrared Control Board

Operating Time or Duty Cycle: 1.000

Complexity (in AEG's): 3

Interaction Factor F: 1.55

Part Type	Quantity N	Failure Rate x 10^6 Unit (^)	Total (N^)	Remarks
Capacitor, Ceramic				
Glass				
Metallized Paper				
Mica				
Paper				
Tantalum, Wet	5	4.0	20.0	
Tantalum, Solid				
Variable				
Unidentified, .001-.1 µ f	9	.05	.45	
Coil, Choke				
Crystal, Quartz				
Diode, Germanium, General				
Rectifier				
Silicon, General	16	.2	3.2	
Rectifier				
Zener	1	.35	.35	
Filter (RLC)				
Fuse				
Relay				
Resistor, Carbon Composition	42	.05	2.1	
Deposited Film				
Potentiometer, CC				
WW				
Rheostat, WW				
Wirewound, Power				
Precision				
Thermistor	3	.3	.9	
Transistor, Germanium, General				
Medium Power				
Power				
Silicon, General	6	.4	2.4	
Medium Power	5	.8	4.0	
Power				
Transformer				
Tube, Electron, Gas Diode				
Pentode				
Triode				
Twin Triode				
Catastrophic Total			33.4	

Several of the results are noteworthy. The subsystem coefficient of variation (0. 507) is fairly large--somewhat larger than might be expected for typical subsystems. This is due to the large individual variance contributions of the several wet-tantalum capacitor types and of some circuits in repetitive use. The largest single contributor to subsystem variance arises from assembly effects. These account for about 43 percent of the total variance, but only for 20 percent of the total failure rate expectation. This result is well in accord with experience and intuition.

An allocation of test effort based on variance would lead to the same decisions obtained from engineering judgment. This does not constitute proof of the validity of the model, but does enhance its plausibility. The example also indicates that the computational effort required is in no way prohibitive.

APPLICATIONS

The model described in this appendix was developed primarily to furnish input data which could be combined with other information for purposes of reliability estimation and test effort allocation. This application has been discussed in detail in the main test of this report. The nature of the model suggests, however, that it can be applied usefully for other purposes.

It usually is possible to construct a functional block diagram for a system in the planning stages, prior to initiation of procurement. Experience makes it possible to assign probable levels of complexity and parts population for individual functional blocks. From these individual blocks, a system reliability estimate can be generated to aid in making judgments as to system feasibility.

The nature of many complex systems, and especially spacecraft systems, is such that the reliability obtainable in single-thread designs and with normal practice does not meet program requirements. Redundancy can be evaluated and applied, but only to the extent permitted by restrictions on available weight and space. Improvements obtainable by other methods at various levels of reliability effort can be provided, but presently available techniques of analysis provide only gross generalizations as to the effectiveness of such methods. Apportionment of reliability requirements among subsystems and at lower levels of assembly is accomplished by crude methods.

By application of the proposed failure rate variability model, the effects of specific reliability improvement measures can be assessed in some detail and in advance. In most instances, good estimates of the "penalties" in terms of cost, increased weight and volume, schedule delay, and power requirements can be associated with these specific reliability improvement measures. This suggests the possibility of exploring, probably with the aid of data processing equipment, many combinations of design and reliability improvement effort as applied to individual units within the system. One might examine a wide variety of combinations and tabulate those which satisfy (or approximate) all major constraints on the total system. This would provide the basis for more effective managerial decisions in choosing among alternative approaches.

The program manager or system prime contractor may then proceed with procurement of system elements to specifications which incorporate the selected parameters as requirements. It is not necessary that the prospective (sub) contractor be advised of the combination of reliability improvement measures associated with the apportioned requirements. In fact, the granting of considerable latitude in the implementation of measures to meet the requirements may be highly desirable. However, it will be necessary to assess each contractor's expected level of achievement based on the procedures actually employed by him; this is the point at which application of the model for its basic purposes is required.

Evaluation forms of the kind illustrated should be applied at this stage. Training of the individuals performing the surveys and evaluation of their results will be required to enhance the validity of the analyses and to assist in future refinement of the model and of model parameter estimates.

In the course of every program, it is anticipated that some of the initial objectives will not be achieved. As these shortcomings are detected by timely application of the proposed approach, reevaluation of apportioned requirements will become necessary. The model employed in the initial apportionment can be employed again for this purpose, subject to additional constraints. As system development progresses, the number of remaining options in system modification decreases. Implementation of additional reliability improvement measures, with accompanying reapportionment of requirements, will remain possible within some number of elements of the system. This can be used to compensate for the observed shortcomings to permit the total system to meet or approach the requirements originally stated for it.

Thus, the model may be applied in initial decision making and apportionment of requirements, in the continuing evaluation of accomplishment throughout the system development and fabrication cycles, and in the rational modification of the system configuration to meet objectives in spite of occurrence of localized deficiencies.

ERRATA

Page Number	Changes
10	Second paragraph, line four, last word: "protion" should read "portion."
26	Second paragraph, first line: "incorporates " should read "incorporate."
28	Second paragraph, line seven: "turn" should read "turns."
36	Last paragraph, second line: "are" should read "is."
	Last paragraph, fifth line: "B" should read "C"
37	First table, second block: "Quality" should read "Quantity."
38	First paragraph, fourth line: should have a comma after "is."
54	Last paragraph, last line: should have a comma after possible.
55	Last paragraph, last line: "chance" should read "change."
Appendix A(4)	Last equation- part of which reads $\left\{\frac{\beta}{1+\beta}\right\}$: the brackets were omitted in the text. The quantity (n + r) is an exponent to which the term in the bracket is raised.

Page Number	Changes
Appendix A(7)	Last paragraph, second line: Insert "in" between first and equality.
Appendix C(9)	First paragraph, fourth line, last word: "virture" should read virtue.
Appendix C(13)	First paragraph, last line: (see reference 1) should read "see footnote on page C(5)."
Appendix C(14)	Last paragraph, second line: to 40 should read : to 4
Appendix C(19)	Under Uses Matched Parts the second and third line : T&C. In each instance the & should be omitted
Appendix C(32)	Last paragraph, fourth line: "test" should read "text".